Mathematics for 'Secondes'

(programme 2000)

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Y. Henel

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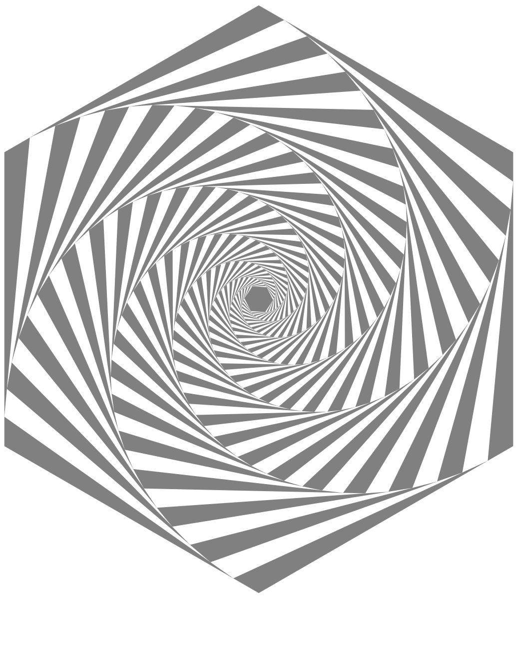
With many thanks to
Donald E. KNUTH who created TeX,
to Leslie LAMPORT, author of LaTeX,
and to all those who provided us
with so much beautifull **free** software.

‘Of all sad words of tongue or pen,’ said he, ‘the saddest are these: “It might have been.” Too late! That is the bitter cry.’

‘In this life, we must be prepared for every emergency. We must distinguish between the unusual and the impossible. Let this be a lesson to you.’

P. G. WODEHOUSE (1881–1975), English writer, in *Mike and Psmith* (1909)



Introduction

Les personnes qui n'ont pas étudié la question sont sujettes à se laisser induire en erreur.

Lord RAGLAN, in *Le Tabou de l'Inceste*, Payot 1935
cité par Boris VIAN (1920–1959), Écrivain français,
in *L'Automne à Pékin* (1956)

Présentation

Cet ouvrage a pour but de fournir à un élève de section européenne « anglais » qui suivrait les mathématiques comme DNL (Discipline Non Linguistique) les bases du vocabulaire anglais spécialisé — couvrant les connaissances acquises au Collège — ainsi qu'un cours complet de 2de conforme au programme applicable à compter de l'année scolaire 2000–2001, paru dans le B.O. Hors-Série Nº 6 du 12 août 1999.

Ce cours ne contient pas, à proprement parlé, d'exercices — on les donnera, tout au long de l'année, dans des fiches thématiques — mais il contient des exemples traités complètement.

Le texte est accompagné de la transcription phonétique des mots spécialisés lorsque le lecteur les rencontre la première fois. Ils sont repris en index. On les retrouvera également dans des tableaux situés en appendices. Les mots y sont rangés par affinité de sens et non pas par ordre alphabétique. Dans ces tableaux on trouvera également la prononciation, les pluriels irréguliers et un équivalent français *fonction* du contexte.

Chapter 1

Numbers

I have often admired the mystical way of Pythagoras[1], and the secret magic of numbers.
Sir Thomas Browne (1605–1682), English writer and physician,
in *Religio Medici* (1643)

1.1 Reminder

1.1.1 Writing and reading numbers

To write numbers one uses ten *figures* [ˈfɪgəʳ] or *digits* [ˈdɪdʒɪt]:

0	zero *or* nought	[ˈzɪərəʊ – nɔːt]	1	one	[wʌn]
2	two	[tuː]	3	three	[θriː]
4	four	[fɔːʳ]	5	five	[faɪv]
6	six	[sɪks]	7	seven	[ˈsevn]
8	eight	[eɪt]	9	nine	[naɪn]

‘Naught’ is a variant spelling (especially in the USA) of ‘nought’.

100 is ‘one hundred’ [ˈhʌndrəd]; 256 is ‘two hundred **and** fifty six’; 7, 547 is ‘seven thousand [ˈθaʊznd] five hundred and forty seven’.

3·142 is ‘three point one four two’. The *decimal point* used to be centered in British English, e.g.[2] 3·14, but not in American English, e.g. 3.14.

[1]Pythagoras [paɪˈθægɒrəs]

[2]e.g. (*abbr.*) ‘for example’, Latin ‘exempli gratia’. It is read [ˌiːˈdʒiː] or [fərɪgˈzɑːmpl].

·472 is 0·472 i.e.[3] 'point four seven two' or 'nought point four seven two'. ·01 is 'nought point nought one' or 'point nought one'.

1.1.2 Scientific Notation — Standard Form

Scientific [ˌsaɪənˈtɪfɪk] notation is a way of writing numbers that makes it easier to work with the very large and small numbers often found in scientific measurements [ˈmeʒəmənt].

When a number is written in scientific notation, the number is written as the base number m (or mantissa [mænˈtɪsə]) times ten raised to a power p (or exponent [ɪkˈspəʊnənt]), expressed as $m \times 10^p$. The mantissa is not less than 1 and less than 10 ($1 \leq m < 10$). The power p is an integer[ˈɪntɪdʒəʳ].

The speed of light expressed in standard form is $2{\cdot}9979 \times 10^8$ m.s^{-1}.

1.1.3 Large Number Names

Number	US Name	British Name
10^6	million	million
10^9	billion	milliard
10^{12}	trillion	billion
10^{15}	quadrillion	1,000 billion
10^{18}	quintillion	trillion
10^{21}	sextillion	1,000 trillion
10^{24}	septillion	quadrillion
10^{27}	octillion	1,000 quadrillion
10^{30}	nonillion	quintillion
10^{33}	decillion	1,000 quintillion

In the 1996 edition of the Oxford English Reference Dictionary it can be read: 'milliard: Now largely superseded by *billion*.'

Around 1484 Nicolas CHUQUET[4] coined the words *billion, trillion, . . . , nonillion*, which also appeared in print in a 1520 book by Émile de la ROCHE. These mathematicians used 'illion' after the prefixes b-, tr-, . . . , non- to denote the 2nd, 3rd, . . . , 9th powers of a million. But around the middle of the 17th century, some other French mathematicians used them instead for the 3rd, 4th, . . . , 10th powers of a thousand.

[3] i.e. (*abbr.*) 'that is to say', Latin 'id est' [aɪd iːst]. It is read [ˌaɪˈiː] or [ˌðætˈɪz]

[4] French mathematician (1445–1500)

Although condemned by the greatest lexicographers as 'erroneous' (LITTRÉ) and 'an entire perversion of the original nomenclature [nəʊˈmenklətʃəʳ] of Chuquet and de la Roche' (MURRAY), the newer usage is now standard in the US, although the older one survives in Britain and is still standard in the continental countries (but the French spelling is nowadays 'llon' rather than 'llion').

1.1.4 Operations

The basic *operations* [ˌɒpəˈreɪʃn] are *addition* [əˈdɪʃn], *subtraction* [səbˈtrækʃn], *multiplication* [ˌmʌltɪplɪˈkeɪʃn] and *division* [dɪˈvɪʒn].

Addition and subtraction

Addition is the operation of combining two numbers to form a *sum* [sʌm]. The symbol for addition is $+$ 'plus [plʌs]', from the Latin, meaning more; it is placed between two numbers to be added together. Thus, $8+7$ means 'eight plus seven' or 'seven added to eight'.

The symbol $=$ means 'is equal [ˈiːkw(ə)l] to' or 'equals', so $8+7=15$ 'eight plus seven equals fifteen'.

Let us consider $a+b=c$. c is the sum, a and b are the summands [ˈsʌˌmænd], addends [əˈdend] or *terms* [tɜːm].

The symbol for subtraction is $-$ 'minus [ˈmaɪnəs]', from the Latin, meaning less; it is placed between two numbers, when the second is to be taken away from the first. For example, $12-9$ means 'twelve minus nine' or 'nine taken away from 12'.

Let us consider $a-b=c$. c is the *difference* [ˈdɪfrəns], a is the minuend [ˈmɪnjʊˌend] and b the subtrahend [ˈsʌbtrəˌhend].

Let a be a number. $-a$ is the *additive inverse* [ˈædɪtɪv ˌɪnˈvɜːs] of a. We have $-a+a=0$.

Multiplication and division

The symbol for multiplication is $\times$ 'times' [taɪmz]. $12\times 5=60$ is read 'twelve times five equals sixty' or 'twelve multiplied [ˈmʌltɪplaɪd] by five equals sixty'.

In $a\times b=c$, a and b are the *factors* [ˈfæktəʳ] and c is the *product* [ˈprɒdʌkt].

The sign for division is $\div$ 'divided [dɪˈvaɪdɪd] by'. Generally we write a/b or $\frac{a}{b}$ 'a over b' instead of $a\div b$.

In $\frac{a}{b} = c$, a is the *dividend* [ˈdɪvɪˌdend], b is the *divisor* [dɪˈvaɪzəʳ] and c is the *quotient* [ˈkwəʊʃnt].

67 when divided by 7 gives a quotient of 9 and a *remainder* [rɪˈmeɪndəʳ] of 4 for we have $67 = 7 \times 9 + 4$.

If a is any number but 0, a has a *multiplicative* [ˌmʌltɪˈplɪkətɪv] *inverse* or *reciprocal* [rɪˈsɪprəkl] which is denoted by $\frac{1}{a}$ or by a^{-1} 'a to the power minus 1'. We have $a \times \frac{1}{a} = 1$.

Factors

Consider $12 = 3 \times 4$; 3 and 4 are said to be *factors* of 12. 12 is said to be a *multiple* [ˈmʌltɪpl] of 3. One can say also that 12 is *exactly divided* or *divisible* [dɪˈvɪzəbl] by 3.

An integer is said to be *even* [ˈiːvn] if and only if it is divisible by 2. If an integer is not even then it is *odd* [ɒd].

A *common factor* of two (or more) numbers is a factor which occurs in both of them (or all of them). The *highest common factor* (H.C.F.) of a group of numbers is the largest number which will divide into all of them.

The *lowest common multiple* (L.C.M.) of two or more numbers is the smallest number into which they will divide exactly.

1.2 The different kinds of numbers

1.2.1 Natural numbers

The *natural numbers* [ˈnætʃrəl ˈnʌmbəʳ] or *whole* [həʊl] *numbers* are 0, 1, 2, 3, ... The set of all natural numbers is denoted by $\mathbb{N}$. To denote that the number n is a natural number one can write $n \in \mathbb{N}$ 'n is in $\mathbb{N}$' *or* 'n belongs to $\mathbb{N}$' *or* 'n is an element [ˈelɪmənt] of $\mathbb{N}$'.

Prime numbers

Many natural numbers have no factors other than themselves and *unity* [ˈjuːnətɪ] (i.e. one), e.g. 7. These numbers are called *prime* [praɪm] *numbers* or *primes*.

A number greater than 1 which is not prime is said to be *composite* [ˈkɒmpəzɪt].

PROPOSITION 1–1:
Every natural number greater than 1 is a product of prime numbers.

For example: $630 = 2 \times 3 \times 3 \times 5 \times 7$.

1.2.2 Integers

The *integers* [ˈɪntɪdʒəʳ] are also called *directed* [dɪˈrektɪd] *numbers* or *signed* [saɪnd] *numbers*. The integers can be obtained as sum or difference of two natural numbers. The set of all the integers is denoted by $\mathbb{Z}$. We have $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

The natural numbers are the positive [ˈpɒzətɪv] integers and 0. So all the elements of $\mathbb{N}$ are also elements of $\mathbb{Z}$. One may write $\mathbb{N} \subset \mathbb{Z}$: 'the set of the natural numbers is a subset of the set of the integers' or 'the set of the natural numbers is included in the set of the integers'.

1.2.3 Rational numbers

A *fraction* [ˈfrækʃn] is the *ratio* [ˈreɪʃɪəʊ] or *quotient* of two numbers. In the fraction $\dfrac{a}{b}$ the number a is said to be the *numerator* [ˈnjuːməreɪtəʳ] and the number b which is not equal to 0 is said to be the *denominator* [dɪˈnɒmɪneɪtəʳ].

A fraction like $\frac{6}{7}$ (written in words 'six sevenths'), in which the numerator is smaller than the denominator, is called a *proper fraction*, whereas one like $\frac{17}{12}$, in which the numerator is greater than the denominator, is called an *improper fraction*. The latter can always be reduced to a whole number — or integer — and a proper fraction, known as a *mixed number*.

Here we have $\dfrac{17}{12} = \dfrac{12+5}{12} = 1 + \dfrac{5}{12}$ which is written $1\dfrac{5}{12}$ 'one and five twelfths'.

$\frac{1}{2}$ is 'one half', $\frac{1}{3}$ 'one third', $\frac{2}{3}$ 'two thirds', $\frac{1}{4}$ 'one quarter', and $\frac{3}{4}$ 'three quarters'.

We can *cancel out* common factors in numerator and denominator, e.g.

$$\frac{24}{28} = \frac{4 \times 6}{4 \times 7} = \frac{6}{7}$$

on dividing out top and bottom by 4.

When the numerator and denominator have no common factor the fraction is said to be 'in its lowest terms'. To reduce a fraction to its lowest terms, divide the numerator and the denominator by their highest common factor.

To find the sum of two unlike fractions — fractions having different denominators — change them to like fractions — fractions having the same denominator — and add the numerators.

To find the quotient of two fractions, multiply the dividend by the inverted [ɪnˈvɜːtɪd] divisor, e.g.

$$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{3} = 1\frac{2}{3}.$$

The *rational numbers* or *rationals* [ˈræʃənl] are the numbers that can be expressed as a ratio or fraction $\frac{a}{b}$ of two integers, a and b, of which the latter may not be zero.

The set of the rationals is denoted by $\mathbb{Q}$.

PROPOSITION 1–2:
For every rational q, there exists one and only one fraction in its lowest terms which is equal to q.

A *decimal fraction* or *decimal* [ˈdesɪməl] is a rational which can be written with a denominator equal to a power of 10. For example, 0.125, $\frac{3}{4}$, and $-\frac{245}{70}$ are decimal for we have:

$$0.125 = \frac{125}{1,000}, \quad \frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75, \quad \text{and} \ -\frac{245}{70} = -\frac{35}{10} = 3.5$$

It is always possible to write an integer as a fraction. So all integers are rationals. We can write $\mathbb{Z} \subset \mathbb{Q}$.

1.2.4 Real numbers

Some numbers are not rational. For instance, π [paɪ] is not a rational. It has been proven[5] that there exists no fraction equal to π. The number π is said to be *irrational* [ɪˈræʃənl]. It is also the case for such numbers as $\sqrt{2}$ 'square root [skweə ruːt] of 2', $\sqrt{3}$ 'square root of 3', ... The rational and irrational numbers together form the set of the *real* [rɪəl] *numbers* or *reals*. The set of the real numbers is denoted by $\mathbb{R}$.

All the rationals are reals and so $\mathbb{Q} \subset \mathbb{R}$.

Let $\mathcal{L}$ be a line [laɪn]. Let O and U be two points [pɔɪnt] on that line such that $OU = 1$. $(O\,;U)$ is said to be a *coordinate* [kəʊˈɔːdnət] *system* of line $\mathcal{L}$.

[5] The proof was given in 1761 by Johann Heinrich LAMBERT (1728–1777), mathematician born in Mülhausen, Germany.

To every point M on $\mathcal{L}$ one can associate [əˈsəʊʃɪeɪt] a real number x_M (x *sub* M), its *abscissa* [æˈbsɪsə][6] which is defined by:

- $x_M = OM$ if M is on the *ray* [reɪ] — or half-line — $[OU)$;
- $x_M = -OM$ if M is not on the ray $[OU)$.

Conversely [ˌkɒnˈvɜːslɪ], to every real number x one can associate on $\mathcal{L}$ a point M the abscissa of which is x. So $\mathbb{R}$ is the set of all the absissae of the points of $\mathcal{L}$.

In such a case $\mathcal{L}$ together with $(\,O\,;\,U\,)$ is said to be the *real axis* [ˈæksɪs].

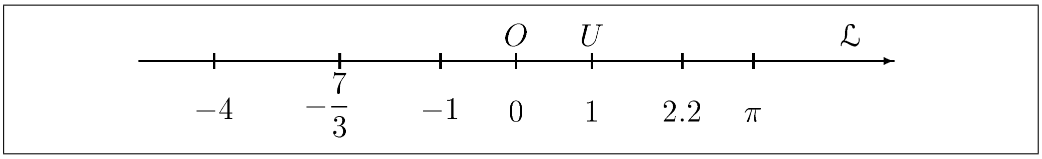


Fig. 1–1

The *positive* [ˈpɒzətɪv] real numbers are the abscissae of the points of $[OU)$ but O. 0 is the abscissa of O. The *negative* [ˈnegətɪv] real numbers are the abscissae of the points which are not on $[OU)$.

$\mathbb{R}^+$ is the set of all the positive real numbers together with 0, in other terms, $\mathbb{R}^+$ is the set of all the non-negative real numbers. $\mathbb{R}^-$ is the set of all the non-positive real numbers. $\mathbb{R}^*$ is the set of all non-null [nʌl] real numbers.

[6] plurals: 'abscissas' [æˈbsɪsəz] or 'abscissae' [æˈbsɪsiː]

Chapter 2

Ordering – absolute value

> Had I been present at the Creation, I would have given some useful hints for the better ordering of the universe.
>
> attributed to Alfonso 'the Wise' (1221–1284)— King of Castile and León from 1252 — on studying the Ptolemaic system.[1]

2.1 Ordering

Two real numbers can always been compared. Let a and b be two real numbers. Then or $a < b$ 'a is less than b' or $a = b$ 'a equals b' or $a > b$ 'a is greater than b'.

If $a < b$ is not true then $a \geq b$ i.e. 'a is not less than b' or 'a is greater than or equal to b'.

If $a > b$ is not true then $a \leq b$ i.e. 'a is not greater than b' or 'a is less than or equal to b'.

Proposition 2–1:
Let a and b be two reals. $a > b$ if and only if $a - b$ is positive; $a \geq b$ if and only if $a - b$ is non-negative; $a < b$ if and only if $a - b$ is negative; and $a \leq b$ if and only if $a - b$ is non-positive.

***Remark**: From now on and for sake of convenience we shall replace the expression 'if and only if' with its common abbreviation 'iff'. We can replace the expression 'if and only if' with 'means' or 'is equivalent to'.*

Let P and Q be propositions — statements, sentences that affirm or deny something

[1]Ptolemaic [ˌtɒlɪˈmeɪɪk]: of or relating to Ptolemy [ˈtɒlɪmɪ], (2nd century), Greek astronomer and geographer.

and which are capable of being true or false. Whenever we know that 'If P then Q' and 'If Q then P' we can state that 'P iff Q'.

As a consequence, for every real a: if $a > 0$ then a is positive; if $a < 0$ then a is negative.

If $a = 0$ then a is said to be *null* [nʌl].

2.1.1 Inequalities

One can add or subtract the same number from both sides of an *inequality* [ˌɪnɪˈkwɒlətɪ] and multiply or divide both sides by positive values without changing the inequality. But when one multiply or divide both sides of the statement by a negative value it is very important that one remembers that the inequality reverses. We can sum this up with:

Proposition 2–2:

Let a, b, and c be reals.

- If $a \leq b$ then $a + c \leq b + c$.
- If $a \leq b$ and $c \geq 0$ then $a \times c \leq b \times c$.
- If $a \leq b$ and $c \leq 0$ then $a \times c \geq b \times c$.

As consequences of that proposition we can state the two following propositions.

Proposition 2–3:

Let a, b, c, and d be reals. If $a \leq b$ and $c \leq d$ then $a + c \leq b + d$.

Proposition 2–4:

Let a, b, c, and d be non-negative reals. If $a \leq b$ and $c \leq d$ then $ac \leq bd$.

2.1.2 Intervals

Definitions

Definition 2–1:

Let a and b be two reals such that $a < b$. The set of real numbers which are not less than a and not greater than b is a *closed interval* [kləʊzd ˈɪntəvl] of $\mathbb{R}$. It is denoted by $[\,a\,;\,b\,]$.

We can write $x \in [\,a\,;\,b\,]$ iff $a \le x \le b$. The numbers a and b are the *endpoints* of the interval $[\,a\,;\,b\,]$. It can be graphically represented as below. NB: we hatch what does not belong to the interval.

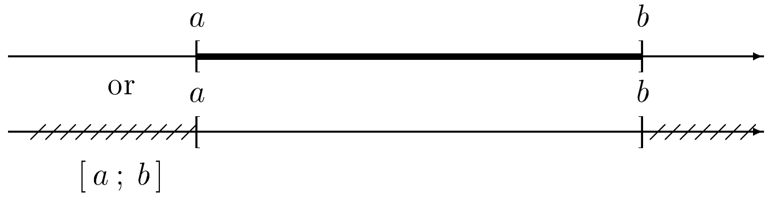


Fig. 2–1

There are nine types of intervals which are:

$$
\begin{aligned}
[\,a\,;\,b\,] &: \text{the set of real } x \text{ such that } a \le x \le b\\
[\,a\,;\,b\,[&: \text{the set of real } x \text{ such that } a \le x < b\\
]\,a\,;\,b\,] &: \text{the set of real } x \text{ such that } a < x \le b\\
]\,a\,;\,b\,[&: \text{the set of real } x \text{ such that } a < x < b\\
[\,a\,;\,+\infty\,[&: \text{the set of real } x \text{ such that } a \le x\\
]\,a\,;\,+\infty\,[&: \text{the set of real } x \text{ such that } a < x\\
]-\infty\,;\,b\,] &: \text{the set of real } x \text{ such that } x \le b\\
]-\infty\,;\,b\,[&: \text{the set of real } x \text{ such that } x < b\\
]-\infty\,;\,+\infty\,[&: \text{the set of all reals}
\end{aligned}
$$

An interval such as $]\,a\,;\,b\,[$ is said to be *open*. An interval such as $[\,a\,;\,b\,[$ is said to be 'closed in a and open in b'. The sign '$+\infty$' is read 'plus infinity' [ɪnˈfɪnətɪ], it does **not** denote a number. The sign '$-\infty$' is read 'minus infinity', it does **not** denote a number either.

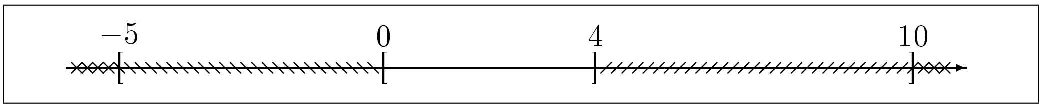
Intersection and union

Definition 2–2:

The *intersection* of two intervals I and J is the set of all reals which are in I **and** in J. It is denoted by $I \cap J$ which is read 'I cap J'.

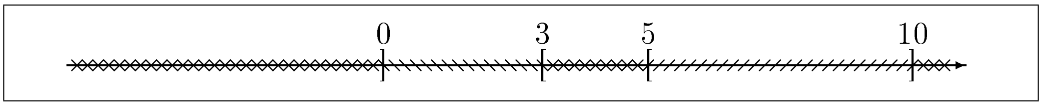
For example, let $I = [\,0\,;\,10\,]$ and $J = [\,-5\,;\,4\,]$, then $I \cap J = [\,0\,;\,4\,]$ for x belongs to $I \cap J$ if and only if $x \in I$ and $x \in J$ that is $0 \le x \le 10$ and $-5 \le x \le 4$ which is equivalent to $0 \le x \le 4$. On the figure below the intersection is represented by the segment which is not hatched.

Fig. 2–2



The intersection of $I = \,]\,0\,;\,3\,[$ and $J = [\,5\,;\,10\,]$ contains no real. One says that this intersection is equal to the *null* [nʌl] *set* (or *void* [vɔɪd] *set* or *empty set*) which is denoted by $\varnothing$. On the figure below all the line is hatched.

Fig. 2–3



The intersection of $I = [\,0\,;\,3\,]$ and $J = [\,3\,;\,10\,]$ contains only 3. One writes $I \cap J = \{3\}$. The intersection of $I = [\,0\,;\,3\,[$ and $J = [\,3\,;\,10\,]$ does not contain any number for 3 does not belong to I. In the last two cases graphical representations are not really useful.

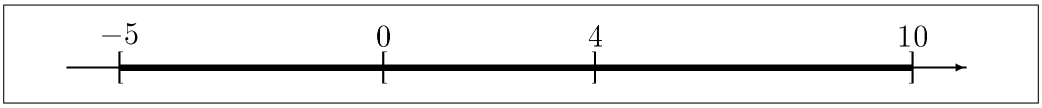
Definition 2–3:

The *union* of two intervals I and J is the set of all reals which are in I or in J. It is denoted by $I \cup J$ which is read 'I cup J'.

For example, let $I = [\,0\,;\,10\,]$ and $J = [\,-5\,;\,4\,]$, then $I \cup J = [\,-5\,;\,10\,]$ for x belongs to $I \cup J$ if and only if $x \in I$ or $x \in J$ that is $0 \le x \le 10$ or $-5 \le x \le 4$ which is equivalent to $-5 \le x \le 10$.

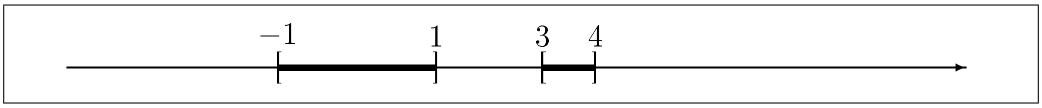
In the following figure each interval is represented with a bold line. The union is then represented by the whole bold segment.

Fig. 2–4



***Remark**: The union of two intervals is not always an interval for a subset of* $\mathbb{R}$ *is an interval iff it is represented by a segment, a ray, or the whole line. For example* $[\,-1\,;\,1\,] \cup [\,3\,;\,4\,]$ *is not an interval.*
The two bold segment do not touch or overlap.

Fig. 2–5



2.2 Distance and absolute value

2.2.1 Definitions

DEFINITION 2–4:

Let x and y be two reals and M and N the points on the real axis the abscissae of which are x and y respectively.
The *distance* [ˈdɪstəns] between x and y, denoted by $d\,(\,x\,;\,y\,)$ is defined by:

$$d\,(\,x\,;\,y\,) = MN.$$

For example $d\,(\,7\,;\,1\,) = 6$ for 6 is the distance between the points with abscissae 1 and 7. For analogous reason $d\,(\,-4\,;\,5\,) = 9$.

Remark: *For any reals x and y, $d\,(\,x\,;\,y\,)$ is non-negative for it equals a distance. Moreover, as $MN = NM$, then $d\,(\,x\,;\,y\,) = d\,(\,y\,;\,x\,)$.*

DEFINITION 2–5:

Let O be the origin on the real axis. Let x be a real and M the point on the real axis the abscissa of which is x. The *absolute value* [ˈæbsəluːt ˈvæljuː] of x, denoted by $|x|$, is the distance from x to 0. It is also the distance from M to O. We have:

$$|x| = d\,(\,0\,;\,x\,) = OM.$$

For example $|3| = 3$ and $|-4| = 4$.

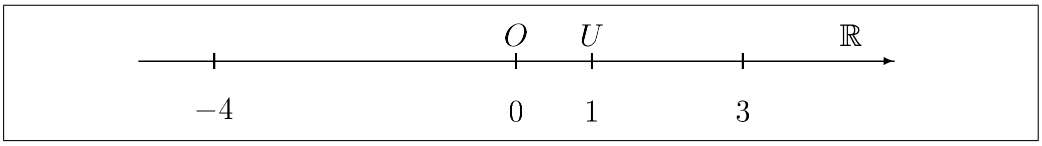


Fig. 2–6

Remark: *For all real x, as $d\,(\,0\,;\,x\,)$ is non-negative, $|x|$ is a non-negative real. For all real x, point M', abscissa $-x$, is the reflection of point M, abscissa x, through O therefore $OM = OM'$ and so $|x| = |-x|$.*

2.2.2 Properties

We now state useful properties some of which are proven. You can try to work out a proof for the remaining ones.

The first one is easy to prove if you remember the geometrical interpretation of the absolute value as a distance on the real axis. (*Draw a figure.*)

PROPOSITION 2–5:
Let x be a real.

- If x is positive, then $|x| = x$.
- If x is negative, then $|x| = -x$.

PROPOSITION 2–6:
For all x and y real numbers.

- If $x \geq y$, then $|x - y| = x - y$.
- If $x \leq y$, then $|x - y| = y - x$.
- $d\,(\,x\,;\,y\,) = |x - y|$.

Proof:
If $x \geq y$ then $x - y \geq 0$ therefore $|x - y| = x - y$.
If $x \leq y$, then $x - y \leq 0$ therefore $|x - y| = -\,(x - y) = y - x$.

PROPOSITION 2–7:
Let α be a non-negative real number and a be any real. For all x real:

$$|x - a| \leq \alpha \quad \text{iff} \quad a - \alpha \leq x \leq a + \alpha \quad \text{iff} \quad x \in [\,a - \alpha\,;\,a + \alpha\,]\,.$$

Proof:
We have to distinguish two cases depending on the place of x relatively to a.
First case: $x \leq a$.
Then $|x - a| \leq \alpha$ is equivalent to $a - x \leq \alpha$ i.e. $a - \alpha \leq x$. Therefore in this case, x fulfills the inequality $|x - a| \leq \alpha$ iff $a - \alpha \leq x \leq a$.
Second case: $x \geq a$.
Then $|x - a| \leq \alpha$ is equivalent to $x - a \leq \alpha$ i.e. $x \leq a + \alpha$. Therefore in this case, x fulfils the inequality $|x - a| \leq \alpha$ iff $a \leq x \leq a + \alpha$.
For all x real, $x \leq a$ or $x \geq a$. So $|x - a| \leq \alpha$ is equivalent to $a - \alpha \leq x \leq a$ or $a \leq x \leq a + \alpha$ i.e. $|x - a| \leq \alpha$ is equivalent to $a - \alpha \leq x \leq a + \alpha$. QED [2]

PROPOSITION 2–8:
Let x and y be two real numbers.

- $|xy| = |x|\;|y|$;
- If $y \neq 0$ then $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$;
- $|x + y| \leq |x| + |y|$ (triangle inequality).

[2] QED stands for latin 'quod erat demonstrandum' and means 'which was to be proven'.

2.3 Approximation

There are basically two types of errors which we need to consider in mathematics: *measurement errors* [ˈmeʒəmənt ˈerəʳ] and *round-off* [ˈraʊndɒf] *errors*. A measurement error is always present whenever a physical [ˈfɪzɪkl] quantity [ˈkwɒntətɪ] is measured [ˈmeʒəd]. A round-off error is present whenever a number is rounded to a specified [ˈspesɪfaɪd] number of decimal places (d.p.) or significant [sɪgˈnɪfɪkənt] figures (s.f.). Round-off errors are also present when a computer *chops off* — or *truncate* [trʌŋˈkeɪt] — numbers which it is working with or displaying. *By* errors *we do not mean* mistakes.

Measuring a value to the nearest unit means deciding that it is nearer one mark than another, in other words it is within half a unit of that mark. For example, if David's age is 14 to the nearest year that means he is actually between $13\frac{1}{2}$ and $14\frac{1}{2}$ and, more precisely, that the age is not less than $13\frac{1}{2}$ and **less than** $14\frac{1}{2}$.

If x is an approximation to an exact number X, then the *absolute error* in x is $e = x - X$. The *absolute error bounds* are the limits between which the absolute error lies. So, if $|e| \leq \varepsilon$, where $\varepsilon > 0$, then ε is called the maximum absolute error bound of x.

Knowing that, we can write $X = x \pm \varepsilon$. That means that the true value of number X lies between $x - \varepsilon$ and $x + \varepsilon$, i.e. $x - \varepsilon \leq X < x + \varepsilon$.

In general, the maximum absolute error bound when a number has been rounded to n decimal places is $\frac{1}{2} \times 10^{-n}$. For example, 'The Sun is 93 million miles from the Earth' means that the Sun is 9.3×10^7 miles from Earth, to the nearest million miles. Here, the absolute error bound is 0.05×10^7 miles i.e. if d is the distance, in miles, from the Earth to the Sun,

$$(9.3 - 0.05) \times 10^7 \leq d < (9.3 + 0.05) \times 10^7.$$

To *round down* a number is to approximate it to a certain number of significant digits or to a whole number or number of tens, hundreds, etc. by replacing the remaining digits by zero. For example, 432.25 can be rounded down to 432.2, 432, 430, or 400 according to circumstances.

To *round up* a number is to approximate it to a certain number of significant digits or to a whole number or number of tens, hundreds, etc. by increasing the relevant digit by one and replacing the remainder by zeros. For example, 486.75 can be rounded up to 486.8, 487, 490, or 500 according to requirements.

Chapter 3

Triangles

3.1 Congruent triangles

Definition 3–1:

Triangles ABC and DEF are said to be *congruent* [ˈkɒngrʊənt] iff

$$AB = DE,\ BC = EF, \text{ and } CA = FD.$$

The sides $[AB]$ and $[DE]$ are said to be *corresponding* [ˌkɒrɪsˈpɒndɪŋ] as are corresponding the vertices A and D and the angles $\angle BAC$ and $\angle EDF$.

This definition will be referred to as the SSS rule (SSS for Side-Side-Side).

On the following figure, triangles ABC, DEF, and GHI are congruent.

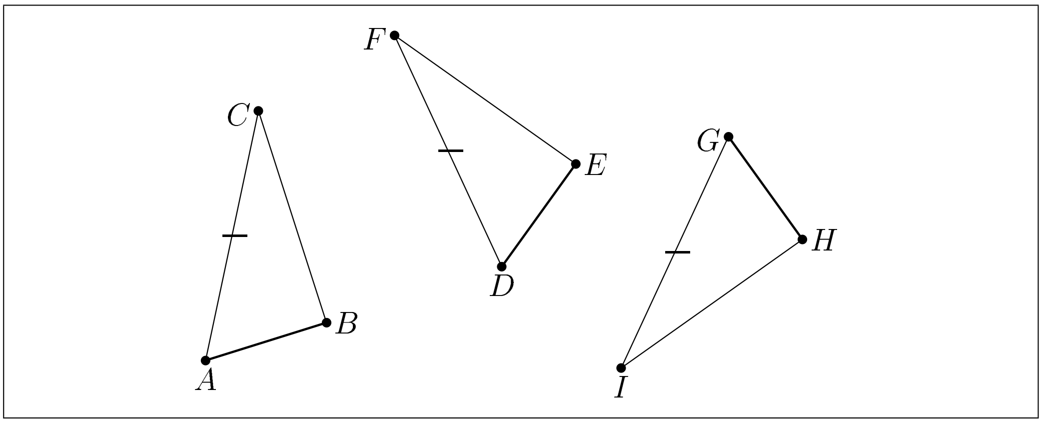


Fig. 3–1

Proposition 3–1:

If ABC and DEF are congruent then their corresponding angles are equal: $\angle BAC = \angle EDF$, $\angle ABC = \angle DEF$, and $\angle ACB = \angle DFE$.

This property is known as the AAA rule (AAA for Angle-Angle-Angle). We can then sum up the preceding property whith: $\boxed{\text{SSS} \Longrightarrow \text{AAA}}$.

PROPOSITION 3–2: (**SAS rule**)
If two sides and the angle between them (the included angle) are the same in both triangles then these triangles are congruent.

The converse of that property is obviously true. So $\boxed{\text{SSS} \Longleftrightarrow \text{SAS}}$.

PROPOSITION 3–3: (**ASA rule**)
If two angles and the side between them are the same in both triangles then these triangles are congruent.

The converse of that property is obviously true. So $\boxed{\text{SSS} \Longleftrightarrow \text{ASA}}$.

PROPOSITION 3–4:
A triangle ABC and its image $A'B'C'$ under an isometry [aɪˈsɒmɪtrɪ] (translation, reflection, rotation) are congruent.

3.2 Similar triangles

DEFINITION 3–2:

Triangles ABC and DEF are said to be *similar* [ˈsɪmɪləʳ] iff

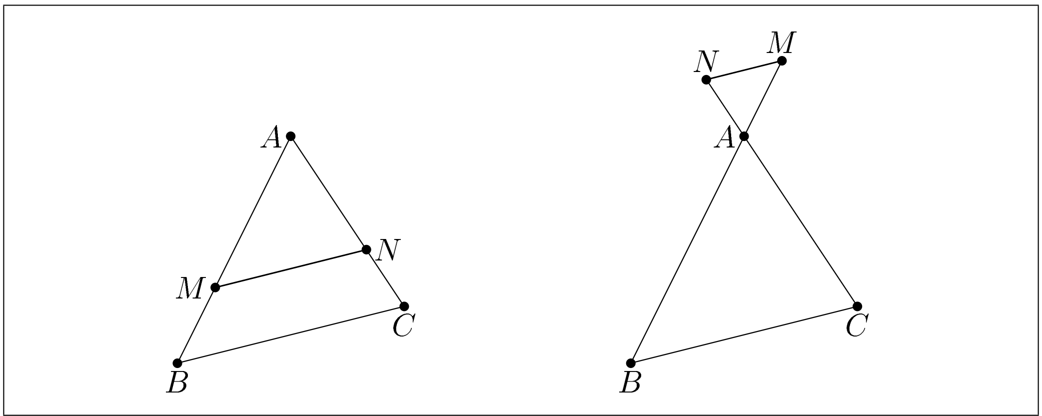
$$\angle BAC = \angle EDF,\ \angle ABC = \angle DEF, \text{ and } \angle ACB = \angle DFE.$$

PROPOSITION 3–5:
If ABC and DEF are similar, then their sides are in the same ratio i.e.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

The sides $[AB]$, $[BC]$, and $[CA]$ are said to be *homologous* [hɒˈmɒləgəs] to the sides $[DE]$, $[EF]$, and $[FD]$ respectively.

Fig. 3–2



PROPOSITION 3–6:

Let ABC be any triangle. Let M and N be points on (AB) and (AC) respectively such that (MN) is parallel to (AB). Then ABC and AMN are similar.

PROPOSITION 3–7:

Let ABC and DEF be triangles such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

Then ABC and DEF are similar.

DEFINITION 3–3:

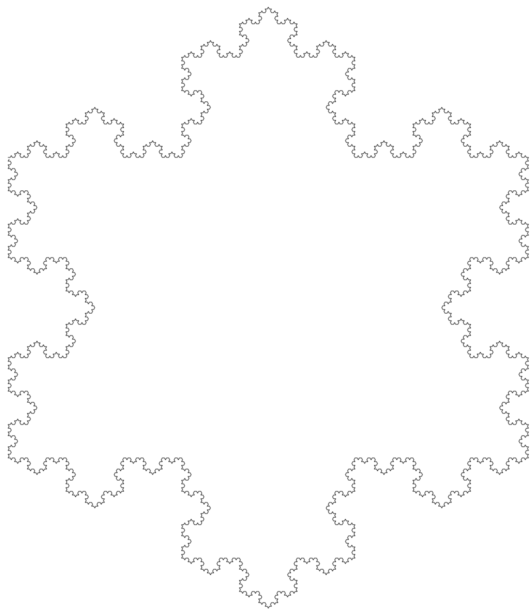
Let ABC and $A'B'C'$ be similar triangles such that

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = k.$$

- If $k = 1$ then the triangles are congruent.
- If $k \neq 1$ then $A'B'C'$ is said to be an *enlargement* of ABC by *scale factor* k.

PROPOSITION 3–8:

If $A'B'C'$ is an enlargement of ABC by scale factor k then the area of $A'B'C'$ is k^2 times the area of ABC.



Chapter 4

Analytic Geometry

> In Geometry (which is the only science that it hath please God hitherto to bestow on mankind) men begin at settling the significations of their words; which . . . they call Definitions.
> Thomas HOBBES (1588–1679), English philosopher,
> in *Leviathan*[1] (1651)

For sake of convenience, we will write $\mathcal{L} \parallel \mathcal{M}$ to denote that the lines $\mathcal{L}$ and $\mathcal{M}$ are parallel.

4.1 Parallelograms

DEFINITION 4–1:

Let A, B, C and D be four points in the plane. $ABCD$ is a *parallelogram* [ˌpæræˈleləgræm] iff its diagonals bisect [baɪˈsekt] each other.

For sake of convenience, we will write $ABCD$ # as an abbreviation of the sentence '$ABCD$ is a parallelogram'.

Parallelograms have many properties of which we state but a few. The first two properties given below involve the four sides of the quadrilateral $ABCD$. The third one involves only two of them.

PROPOSITION 4–1:

$ABCD$ is a parallelogram iff its opposite sides are parallel, i.e. iff $(AB) \parallel (CD)$ and $(AD) \parallel (BC)$.

[1] Hobbes [hɒbz] — leviathan [lɪˈvaɪəθn]

Proposition 4–2:
$ABCD$ is a parallelogram iff its opposite sides have the same length, i.e. iff $AB = CD$ and $AD = BC$.

Proposition 4–3:
$ABCD$ is a parallelogram iff two opposite sides are parallel and have equal lengths, i.e. iff e.g. $(AB) \parallel (CD)$ and $AB = CD$.

4.2 Vectors

4.2.1 Equality

Let A and B be two points. The vector $\overrightarrow{AB}$ is represented by an arrow from A to B. A is the *beginning point* and B the *end point*. If $\overrightarrow{AB} = \overrightarrow{CD}$ we can decide to denote that vector with a single sign as e.g. $\overrightarrow{u}$. So $\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{u}$. In such a case the *ordered pairs* $(A\,;\,B)$ and $(C\,;\,D)$ are said to represent the vector $\overrightarrow{u}$.

In English texts one can find **u** or $\underline{u}$ instead of $\overrightarrow{u}$ to denote the vector 'u'.

Definition 4–2:

The *magnitude* [ˈmagnɪtjuːd] (or *length* or *modulus* [ˈmɒdjʊləs]) of the vector $\overrightarrow{AB}$ is the length of $[AB]$ i.e. AB. The magnitude of a vector $\overrightarrow{u}$ is denoted by $\lVert\overrightarrow{u}\rVert$.

Definition 4–3:

Let A, B, C and D be four points in the plane. The vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal iff $ABDC$ is a parallelogram. $$\overrightarrow{AB} = \overrightarrow{CD} \iff ABDC\ \#$$

***Remark**: If $\overrightarrow{AB} = \overrightarrow{CD}$ we can say that those vectors have the same* direction *— $(AB) \parallel (CD)$ — the same* sense *— B and D are on the same side of (AC) — and the same length.*

Proposition 4–4:
$\overrightarrow{AB} = \overrightarrow{CD}$ iff D is the image under $t_{\overrightarrow{AB}}$ of C.

Proposition 4–5:
Let $\overrightarrow{u}$ be any vector and let A be any point in the plane. There exists one and only one point B such that $\overrightarrow{u} = \overrightarrow{AB}$.

So one can always represent any given vector with a given beginning point. Obviously one can also represent any given vector with a given end point.

4.2.2 Sum

Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two vectors. There are two ways of defining the sum of the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ depending on how they are represented.

PROPOSITION 4–6: (**Triangle law**)

If A, B and C are points such that $\overrightarrow{u} = \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{BC}$ then

$$\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}.$$

PROPOSITION 4–7: (**Parallelogram law**)

If A, B and C are points such that $\overrightarrow{u} = \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{AC}$. Let D be the point such that $CABD$ is a parallelogram. Then

$$\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}.$$

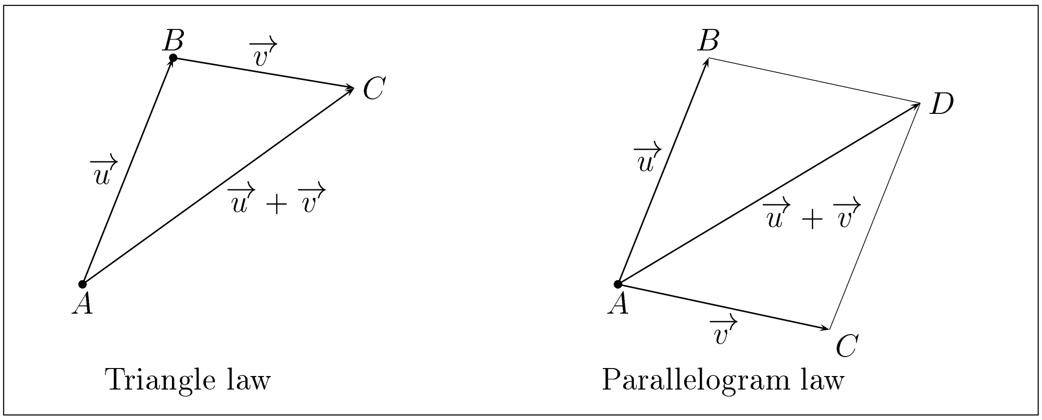


Fig. 4–1

In fact if $CABD$ is a parallelogram,

$$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BD}$$

and we can conclude using the triangle law[2].

DEFINITION 4–4:

The *null vector* $\overrightarrow{0}$ is equal to $\overrightarrow{AA}$ for any point A. For any vector $\overrightarrow{u}$:

$$\overrightarrow{u} + \overrightarrow{0} = \overrightarrow{0} + \overrightarrow{u} = \overrightarrow{u}.$$

[2] which is known in French as 'la relation de Chasles'.

DEFINITION 4–5:

$\overrightarrow{v}$ is said to be the *inverse* of $\overrightarrow{u}$ iff

$$\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u} = \overrightarrow{0}.$$

We denote the (additive) inverse of $\overrightarrow{u}$ by $-\overrightarrow{u}$.

***Remark**: In French one must use the word 'opposé'.*

If A and B are such that $\overrightarrow{u} = \overrightarrow{AB}$ then $-\overrightarrow{u} = \overrightarrow{BA}$ for

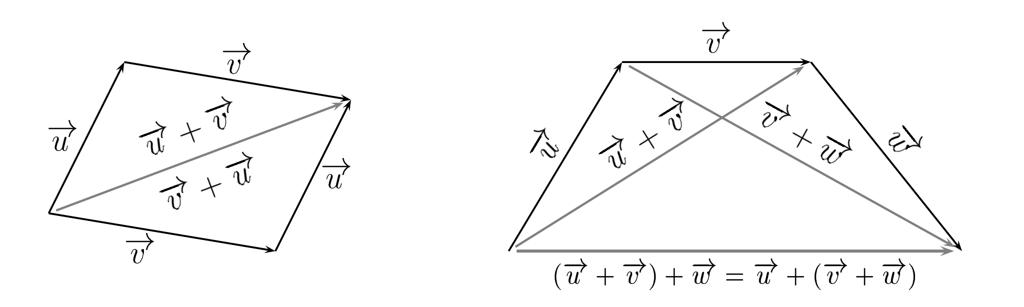
$$\overrightarrow{AB} + \overrightarrow{BA} = \overrightarrow{AA} = \overrightarrow{0} \text{ and } \overrightarrow{BA} + \overrightarrow{AB} = \overrightarrow{BB} = \overrightarrow{0}.$$

PROPOSITION 4–8: (**Properties of the sum**)

Let $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ be vectors.

- $(\overrightarrow{u} + \overrightarrow{v}) + \overrightarrow{w} = \overrightarrow{u} + (\overrightarrow{v} + \overrightarrow{w})$;
- $\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}$;

Fig. 4–2



4.2.3 Multiplication by a scalar

***Remark**: When dealing with vectors, any real number is called a* scalar [ˈskeɪləʳ].

If $\overrightarrow{AB} = \overrightarrow{u}$, we know how to represent $\overrightarrow{u} + \overrightarrow{u}$, $\overrightarrow{u} + \overrightarrow{u} + \overrightarrow{u}$ or even $\overrightarrow{u} + \overrightarrow{u} + \overrightarrow{u} + \overrightarrow{u}$ and it would be agreable to be allowed to write $2\overrightarrow{u}$, $3\overrightarrow{u}$ and $4\overrightarrow{u}$ instead of those rather boring long sums. That will be the case at the end of this section the aim of which is to define the multiplication of a vector by a scalar (or number).

DEFINITION 4–6:

Let $\overrightarrow{u}$ be a vector and k be a number. We define the product of $\overrightarrow{u}$ by k, denoted by $k\overrightarrow{u}$, as follows:

- If $\overrightarrow{u} = \overrightarrow{0}$ or $k = 0$ then $k\overrightarrow{u} = \overrightarrow{0}$.
- If $\overrightarrow{u} \neq \overrightarrow{0}$ and $k > 0$ then $k\overrightarrow{u}$ has the same direction and sense as $\overrightarrow{u}$ and $\|k\overrightarrow{u}\| = k \times \|\overrightarrow{u}\|$.
- If $\overrightarrow{u} \neq \overrightarrow{0}$ and $k < 0$ then $k\overrightarrow{u}$ has the same direction and sense as $-\overrightarrow{u}$ and $\|k\overrightarrow{u}\| = -k \times \|\overrightarrow{u}\|$.

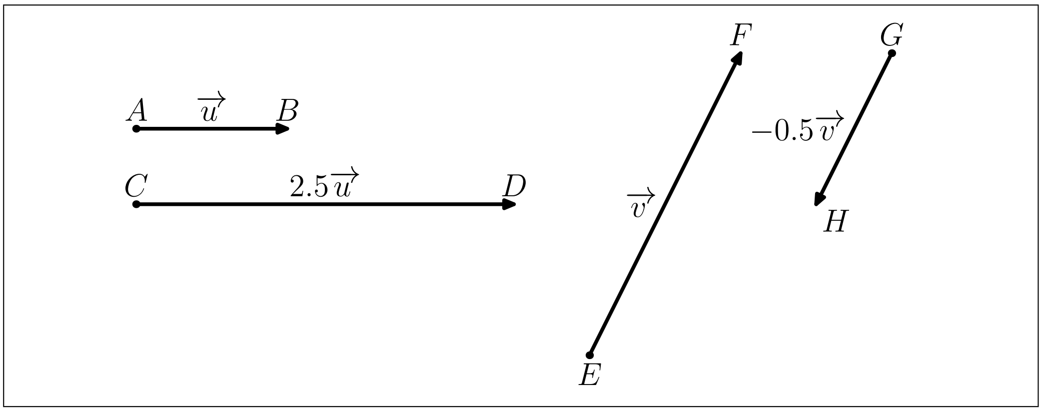


Fig. 4–3

DEFINITION 4–7:

Vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are said to be *parallel* iff there exists a scalar k such that $\overrightarrow{u} = k\overrightarrow{v}$ or $\overrightarrow{v} = k\overrightarrow{u}$.

When necessary we will write $\overrightarrow{u} \parallel \overrightarrow{v}$ to denote that vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are parallel.

***Remark**: In French one must use the word 'colinéaire' in such a case.*

For each vector $\overrightarrow{u}$ the following equality holds: $\overrightarrow{0} = 0\overrightarrow{u}$. So $\overrightarrow{0}$ is parallel to any vector.

PROPOSITION 4–9:

Let A, B, C and D be four distinct points.

- $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel iff (AB) and (CD) are parallel;
- $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel iff C is on (AB).

The last proposition is very useful to prove that three points are collinear.

PROPOSITION 4–10: (**Properties of the multiplication by a scalar**)
Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be vectors and a and b be scalars.

- $a\,(\overrightarrow{u} + \overrightarrow{v}) = a\overrightarrow{u} + a\overrightarrow{v}$;
- $(a+b)\,\overrightarrow{u} = a\overrightarrow{u} + b\overrightarrow{u}$;
- $a\,(b\overrightarrow{u}) = (a \times b)\,\overrightarrow{u}$;
- $0\overrightarrow{u} = \overrightarrow{0}$;
- $(-1)\overrightarrow{u} = -\overrightarrow{u}$.

4.2.4 Basis

DEFINITION 4–8:

The ordered pair of vectors $(\,\overrightarrow{u}\,;\,\overrightarrow{v}\,)$ is said to be a *basis*[3] [ˈbeɪsɪs] iff $\overrightarrow{u}$ and $\overrightarrow{v}$ are not parallel.

Question: Let A and C be two points, $\mathcal{L}$ and $\mathcal{L}'$ be two intersecting lines. Determine B and D such that $ABCD$ is a parallelogram, (AB) is parallel to $\mathcal{L}$ and (AD) is parallel to $\mathcal{L}'$.

Answer: *B is the common point of the line parallel to $\mathcal{L}$ passing through A and the line parallel to $\mathcal{L}'$ passing through C. Similarly, D is the common point of the line parallel to $\mathcal{L}'$ passing through A and the line parallel to $\mathcal{L}$ passing through C.*

If $(\,\overrightarrow{u}\,;\,\overrightarrow{v}\,)$ is a basis, there are three non-collinear points O, U and V such that $\overrightarrow{u} = \overrightarrow{OU}$ and $\overrightarrow{v} = \overrightarrow{OV}$. In that case, (OU) and (OV) intersect. Let $\overrightarrow{w}$ be a vector. There are two points A and C such that $\overrightarrow{w} = \overrightarrow{AC}$. Applying the preceding result, we can construct $ABCD$, parallelogram such that $(AB) \parallel (OU)$ and $(AD) \parallel (OV)$. Then $\overrightarrow{AB} \parallel \overrightarrow{OU}$ and there exists one (and only one) number x such that $\overrightarrow{AB} = x\,\overrightarrow{OU}$. By the same reasoning we obtain a number y such that $\overrightarrow{AD} = y\,\overrightarrow{OV}$. Then using the parallelogram law we prove that $\overrightarrow{w} = x\overrightarrow{u} + y\overrightarrow{v}$.

That result is important and we restate it in the following proposition.

[3] *plur.* bases [ˈbeɪsiːz]

PROPOSITION 4–11:
If $(\overrightarrow{u}\,;\,\overrightarrow{v})$ is a basis, for any $\overrightarrow{w}$, there exist two numbers x and y such that $\overrightarrow{w} = x\overrightarrow{u} + y\overrightarrow{v}$.
Moreover if $\overrightarrow{w} = x\overrightarrow{u} + y\overrightarrow{v} = a\overrightarrow{u} + b\overrightarrow{v}$ then $x = a$ and $y = b$.

DEFINITION 4–9:

If $(\overrightarrow{u}\,;\,\overrightarrow{v})$ is a basis, the *coordinates* of a vector $\overrightarrow{w}$ are the numbers x and y such that $\overrightarrow{w} = x\overrightarrow{u} + y\overrightarrow{v}$.
We write $\overrightarrow{w} : (x\,;\,y)$.

Coordinates are sometimes written in a column like this $\begin{pmatrix} x \\ y \end{pmatrix}$.

Let $(\overrightarrow{\imath}\,;\,\overrightarrow{\jmath})$ be a basis, let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two vectors the coordinates of which are $(u\,;\,u')$ and $(v\,;\,v')$ respectively, and let a be a number.

$$\begin{aligned}\overrightarrow{u} + \overrightarrow{v} &= (u\overrightarrow{\imath} + u'\overrightarrow{\jmath}) + (v\overrightarrow{\imath} + v'\overrightarrow{\jmath}) \\ &= (u+v)\overrightarrow{\imath} + (u'+v')\overrightarrow{\jmath}, \text{and} \\ a\overrightarrow{u} &= a\,(u\overrightarrow{\imath} + u'\overrightarrow{\jmath}) = au\overrightarrow{\imath} + au'\overrightarrow{\jmath}.\end{aligned}$$

Therefore we can state the proposition:

PROPOSITION 4–12:
Let $\overrightarrow{u} : (u\,;\,u')$ and $\overrightarrow{v} : (v\,;\,v')$.

- $\overrightarrow{u} = \overrightarrow{0}$ iff $u = 0$ and $u' = 0$;
- $\overrightarrow{u} = \overrightarrow{v}$ iff $u = v$ and $u' = v$;
- $\overrightarrow{u} + \overrightarrow{v} : (u+v\,;\,u'+v')$;
- for each number k, $k\overrightarrow{u} : (ku\,;\,ku')$.

DEFINITION 4–10:

Let $(\overrightarrow{\imath}\,;\,\overrightarrow{\jmath})$ be a basis and O be a point in the plane. The ordered triple [trɪpl] $(O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath})$ is said to be a Cartesian[4] *coordinate system.*
Let M be a point in the plane. The coordinates of M — relatively to the coordinate system $(O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath})$ — are $(x\,;\,y)$ iff the coordinates of $\overrightarrow{OM}$ are $(x\,;\,y)$ in the basis $(\overrightarrow{\imath}\,;\,\overrightarrow{\jmath})$.

PROPOSITION 4–13:
If A and B have coordinates $(a\,;\,a')$ and $(b\,;\,b')$ respectively then $\overrightarrow{AB}$ has coordinates $(b-a\,;\,b'-a')$.

[4]Cartesian [kɑːˈtiːziːən] of or relating to René DESCARTES (1596–1650) or his work in philosophy, science, and mathematics.

Proposition 4–14: (**Parallelism criterion**)
Let $(\vec{\imath}\,;\,\vec{\jmath})$ be a basis, $\vec{u}$ and $\vec{v}$ two vectors the respective coordinates of which are $(u\,;\,u')$ and $(v\,;\,v')$.
$\vec{u}$ and $\vec{v}$ are parallel iff $uv' - u'v = 0$.

***Remark**: Criterion* [kraɪˈtɪərɪən] plur. *criteria* [kraɪˈtɪərə]*: a principle or standard that a thing is judged by.*

The parallelism criterion can be used to determine when a given point is on a given line. Let $(O\,;\,\vec{\imath}\,;\,\vec{\jmath})$ be a coordinate system.

Example:

1. Let C be the point the coordinates of which are $(1\,;\,3)$. Let $\mathcal{L}$ be the line passing through A: $(-1\,;\,2)$ and B: $(5\,;\,-1)$. Is C on $\mathcal{L}$? In fact, C is on $\mathcal{L}$ if and only if $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. We can determine the coordinates of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and then use the parallelism criterion (cf. 4–14) to answer the question.

   $\overrightarrow{AB}$: $(5-(-1)\,;\,-1-2)$ so $\overrightarrow{AB}$: $(6\,;\,-3)$ and $\overrightarrow{AC}$: $(1-(-1)\,;\,3-2)$ so $\overrightarrow{AC}$: $(2\,;\,1)$. Now as $6\times 1-(-3)\times 2 = 12$, we can state that C is **not** on $\mathcal{L}$.

2. Let A: $(5\,;\,-15)$, B: $(3\,;\,-8)$, and C: $(1\,;\,-1)$. The question is once more: is C on (AB)?

   We have $\overrightarrow{AB}$: $(-2\,;\,7)$ and $\overrightarrow{AC}$: $(-4\,;\,14)$. As $-2\times 14-7\times(-4) = -28+28 = 0$ we can state that C **is** on (AB).

3. Let A: $(5\,;\,-15)$, B: $(3\,;\,-8)$ once again. Let C be a point of (AB) the abscissa of which is 12. What is the ordinate of C?

   Let y be the (unknown) ordinate of C. $\overrightarrow{AB}$: $(-2\,;\,7)$ and $\overrightarrow{AC}$: $(7\,;\,y+15)$. As C is on (AB), $-2\times(y+15)-7\times 7 = 0$ that is $-2y-79 = 0$ and then $y = -39.5$.

4.2.5 Midpoint

One knows that the midpoint [ˈmɪdpɔɪnt] of $[AB]$ is the point I such that I is on (AB) and is equidistant from A and B.

Proposition 4–15:
I is the midpoint of $[AB]$ iff one of the following equalities holds:

- $\overrightarrow{AI} = \overrightarrow{IB}$;
- $\overrightarrow{AI} = \frac{1}{2}\overrightarrow{AB}$ or $\overrightarrow{IB} = \frac{1}{2}\overrightarrow{AB}$;

- $\overrightarrow{AB} = 2\overrightarrow{AI}$ or $\overrightarrow{AB} = 2\overrightarrow{IB}$;
- $\overrightarrow{IA} + \overrightarrow{IB} = \overrightarrow{0}$.

PROPOSITION 4–16:
If A and B have coordinates $(a\,;\,a')$ and $(b\,;\,b')$ respectively then the midpoint of $[AB]$ has coordinates $\left(\dfrac{a+b}{2}\,;\,\dfrac{a'+b'}{2}\right)$.

PROPOSITION 4–17:
If I is the midpoint of $[AB]$ then for any point M $\quad\overrightarrow{MA} + \overrightarrow{MB} = 2\overrightarrow{MI}$.

That is easily proved for

$$\begin{aligned}\overrightarrow{MA} + \overrightarrow{MB} &= \overrightarrow{MI} + \overrightarrow{IA} + \overrightarrow{MI} + \overrightarrow{IB}\\ &= 2\,\overrightarrow{MI} + \overrightarrow{IA} + \overrightarrow{IB}\\ &= 2\,\overrightarrow{MI} + \overrightarrow{0} = 2\,\overrightarrow{MI} \quad \text{QED.}\end{aligned}$$

PROPOSITION 4–18:
If I and J are the midpoints of $[AB]$ and $[AC]$ respectively then $\overrightarrow{IJ} = \dfrac{1}{2}\overrightarrow{BC}$.

4.3 Equations of lines

Let $(O\,;\,\vec{\imath}\,;\,\vec{\jmath})$ be a coordinate system in the plane.

DEFINITION 4–11:

Let $\vec{u}$ be a vector and $\mathcal{L}$ be a line. $\vec{u}$ is a *direction vector* of $\mathcal{L}$ iff there exist two points A and B on $\mathcal{L}$ such that $\vec{u} = \overrightarrow{AB}$.

PROPOSITION 4–19:
If $\vec{u}$ is a direction vector of $\mathcal{L}$ then, for any real k different from 0, $k\,\vec{u}$ is also a direction vector of $\mathcal{L}$.

Let A and B be two points. Two cases occur: or x_A equals x_B or not.

In the first case, $\overrightarrow{AB}$ has coordinates $(0\,;\,y_B - y_A)$. Let M be a point with coordinates $(x\,;\,y)$. M is on (AB) if and only if $\overrightarrow{AM}$ and $\overrightarrow{AB}$ are parallel i.e. $(y_B-y_A)(x-x_A)-0\times(y-y_A) = 0$ which is equivalent to $x = x_A$.

In that case, line (AB) is parallel to the y-axis.

In the second case, $\overrightarrow{AB}$ has coordinates $(x_B - x_A\,;\,y_B - y_A)$ and $x_B - x_A \neq 0$. Let M be a point with coordinates $(x\,;\,y)$. M is on (AB) if and

only if $\overrightarrow{AM}$ and $\overrightarrow{AB}$ are parallel i.e.
$(y_B - y_A) \times (x - x_A) - (x_B - x_A) \times (y - y_A) = 0$
which is equivalent to
$(y_B - y_A)x - (x_B - x_A)y + ((x_B - x_A)y_A - (y_B - y_A)x_A) = 0$
and as that equation characterises the fact that a point is on (AB) or not, it is named ‘equation of (AB)’.

That equation is equivalent to
$(x_B - x_A)y = (y_B - y_A)x + ((x_B - x_A)y_A - (y_B - y_A)x_A) = 0$, and as $x_B - x_A \neq 0$, one can write

$$y = \frac{y_B - y_A}{x_B - x_A}x + \frac{(x_B - x_A)y_A - (y_B - y_A)x_A}{x_B - x_A}$$

which is also an equation of (AB).

In such an equation the coefficient of x is called the *slope* [sləʊp] or *gradient* [ˈgreɪdɪənt] of (AB). The number $\frac{(x_B - x_A)y_A - (y_B - y_A)x_A}{x_B - x_A}$ is called the *intercept*. It is the ordinate of the point at which (AB) cuts the y-axis. The last equation is said to have the *slope-intercept form*.

We can sum that up with:

Proposition 4–20:
Let $A : (\, x_A \,;\, y_A \,)$ and $B : (\, x_B \,;\, y_B \,)$ be two points in the plane.

- If $x_A = x_B$ then (AB) has equation $x = x_A$.
- If $x_A \neq x_B$ then the slope of (AB) is $m = \dfrac{y_B - y_A}{x_B - x_A}$

If a line $\mathcal{L}$ is parallel to the y-axis then all its points have the same abscissa. That is why such a line has equation $x = k$ for a certain real number k.

If contrariwise $\mathcal{L}$ is not parallel to the y-axis, then there exist two points which have different abscissae and — it is the second case stated above — it is the possible to find two real numbers m and p such that $\mathcal{L}$ has equation $y = mx + p$. The points A: $(\, 0 \,;\, p \,)$ and B: $(\, 1 \,;\, m + p \,)$ are on $\mathcal{L}$ and so the vector $\overrightarrow{u}$: $(\, 1 \,;\, m \,)$ is a direction vector of $\mathcal{L}$.

We can now state:

Proposition 4–21: (**Equation of a line**)
1 Each line parallel to the y-axis has an equation of the form $x = a$ $(a \in \mathbb{R})$ where a is the abscissa of every point on the line.

Each line intersecting the y-axis has an equation of the form $y = mx + p$ with m and p two real numbers.

2 The set of the points $M : (x\,;\,y)$ such that:

- $x = k$ with k a real, is a line parallel to the y-axis.
- $y = mx+p$ is a line which passes through the point with coordinates $(0\,;\,p)$ and has $\overrightarrow{d} : (1\,;\,m)$ as direction vector.

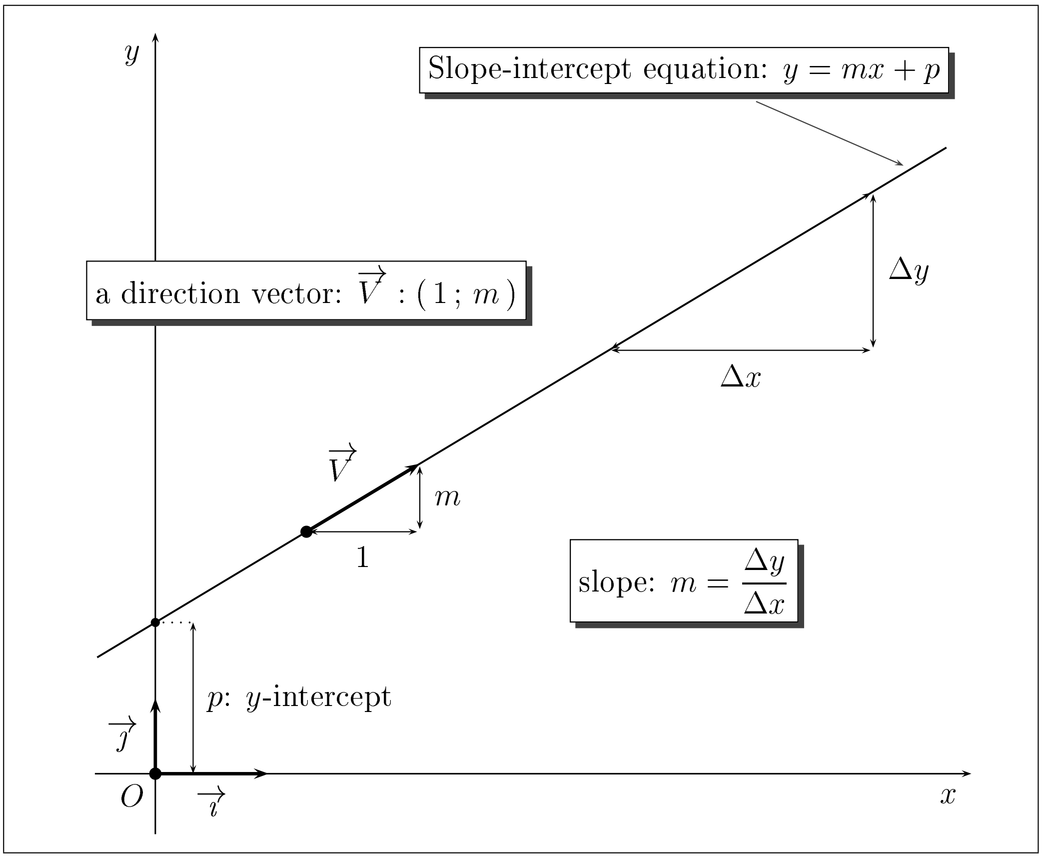


Fig. 4–4

Let a, b and c be real numbers such that a and b are not simultaneously null. Consider the equation $\mathcal{E}$: $ax + by + c = 0$. First consider that $b = 0$. Then $a \neq 0$ and so $\mathcal{E}$ is equivalent to $x = -\dfrac{c}{a}$ so it is an equation of a line parallel to the y-axis. Then $(0\,;\,1)$ is a direction vector of that line and so is $(0\,;\,a)$ for a is not null.

If $b \neq 0$ then $\mathcal{E}$ is equivalent to $y = -\dfrac{a}{b}x - \dfrac{c}{b}$. So $\mathcal{E}$ is an equation of line with direction vector $\left(1\,;\,-\dfrac{a}{b}\right)$. We can multiply that vector by $-b$ to obtain an other direction vector, for b is not null, and we can state the following proposition.

PROPOSITION 4–22:
Let a, b and c be three reals such that a and b are not simultaneously null. The set of the points $M : (\, x\, ;\, y\,)$ such that $ax + by + c = 0$ is a line with direction vector $\overrightarrow{V} : (\, -b\, ;\, a\,)$.

If two lines are parallel they have the same direction vectors. So if $(\, 1\, ;\, p\,)$ is a direction vector of the first then it is also a direction vector of the second. If a line does not intersect the y-axis it has a slope p and direction vector $(\, 1\, ;\, p\,)$. Therefore:

PROPOSITION 4–23:
Let $\mathcal{L}$ and $\mathcal{L}'$ be two lines intersecting the y-axis. $\mathcal{L}$ and $\mathcal{L}'$ are parallel iff they have the same slope.

4.4 Orthonormal bases and coordinate systems

DEFINITION 4–12:

The vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are *orthogonal* [ɔːˈθɒgənl] iff one of the following conditions holds:

1. $\overrightarrow{u}$ or $\overrightarrow{v}$ is equal to $\overrightarrow{0}$;
2. any line with direction vector $\overrightarrow{u}$ is perpendicular to any line with direction vector $\overrightarrow{v}$.

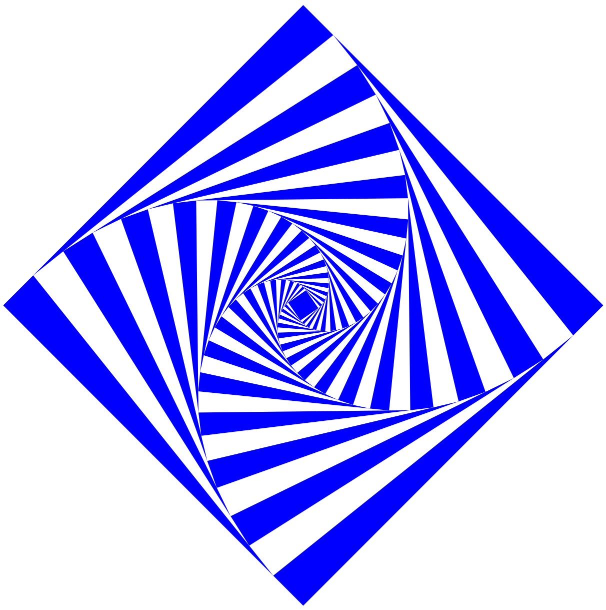
One can write $\overrightarrow{u} \perp \overrightarrow{v}$ to denote the fact that $\overrightarrow{u}$ and $\overrightarrow{v}$ are orthogonal.

DEFINITION 4–13:

- The basis $(\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$is said to be *orthogonal* iff $\overrightarrow{\imath} \perp \overrightarrow{\jmath}$.
- The coordinate system $(\, O\, ;\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$ is said to be orthogonal iff $(\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$is an orthogonal basis.
- The basis $(\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$is said to be *orthonornal* [ɔːθəˈnɔːm(ə)l] iff $\overrightarrow{\imath} \perp \overrightarrow{\jmath}$ and $\|\overrightarrow{\imath}\| = \|\overrightarrow{\jmath}\| = 1$.
- The coordinate system $(\, O\, ;\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$ is said to be orthonormal iff $(\, \overrightarrow{\imath}\, ;\, \overrightarrow{\jmath}\,)$is an orthonormal basis.

Proposition 4–24:
Let $\left(O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath}\right)$ be an orthonormal coordinate system.

- Let $A : (\, x_A \,;\, y_A \,)$ and $B : (\, x_B \,;\, y_B \,)$ be two points in the plane. Then
$$AB = \left\| \overrightarrow{AB} \right\| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}.$$
- Let $\overrightarrow{u}$: $(\, x \,;\, y \,)$. Then $\|\overrightarrow{u}\| = \sqrt{x^2 + y^2}$.



Chapter 5

Transformations

> Beauty is the first test: there is no permanent place in the world for ugly mathematics.
> Godfrey Harold HARDY (1877–1947), English mathematician, in *A Mathematician's Apology* (1940)

A *transformation* [ˌtrænsfəˈmeɪʃn] of the plane is a one-to-one mapping of points of the plane onto points of the plane. It is a kind of process which associates to each point M in the plane one and only point M' in the plane. M' is the *image* of M under the transformation; M is the *pre-image* [priːˈɪmɪdʒ] of M'. Transformations are one-to-one because every point M' has one and only one pre-image under a transformation.

5.1 Translations and reflections

5.1.1 Translations

DEFINITION 5–1:

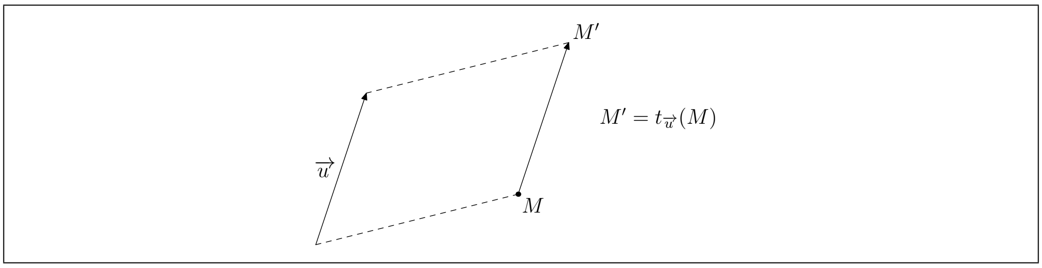
The *translation* [trænsˈleɪʃn] by *vector* [ˈvektəʳ] $\overrightarrow{u}$ is the transformation which to each point M in the plane associates the point M' such that

$$\overrightarrow{MM'} = \overrightarrow{u}.$$

The translation by $\overrightarrow{u}$ is denoted by $t_{\overrightarrow{u}}$ (read 't sub u') or by t whenever there is no ambiguity. The image of the point M under $t_{\overrightarrow{u}}$ is denoted by $t_{\overrightarrow{u}}(M)$ (read 't sub u of M').

***Remark**: The image of the figure* $\mathcal{S}$ *under* $t_{\overrightarrow{u}}$ *is denoted by* $t_{\overrightarrow{u}}(\mathcal{S})$, *whereas the image of the line* (AB) *under* $t_{\overrightarrow{u}}$ *is denoted by* $t_{\overrightarrow{u}}(AB)$ *to avoid the more cumbersome, even if more correct,* $t_{\overrightarrow{u}}((AB))$.

Fig. 5–1



PROPOSITION 5–1:

Let A and B be two points in the plane. Let $\overrightarrow{u}$ be a vector and t be the translation by $\overrightarrow{u}$. Let $A' = t(A)$ and $B' = t(B)$.

- $\overrightarrow{A'B'} = \overrightarrow{AB}$ so $A'B' = AB$;
- $t(AB) = (A'B')$ and $(A'B')$ is parallel to (AB);
- if $\overrightarrow{u} \neq \overrightarrow{0}$ then there is no *invariant* [ɪnˈveərɪənt] point under t i.e. for all point M in the plane $t(M) \neq M$.

***Remark**: To each point* M *the translation by* $\overrightarrow{0}$ *(null vector) associates* M *itself. All the points in the plane are invariant. This translation is said to be the* identity *[*aɪˈdentətɪ*] of the plane and usually denoted by* Id.

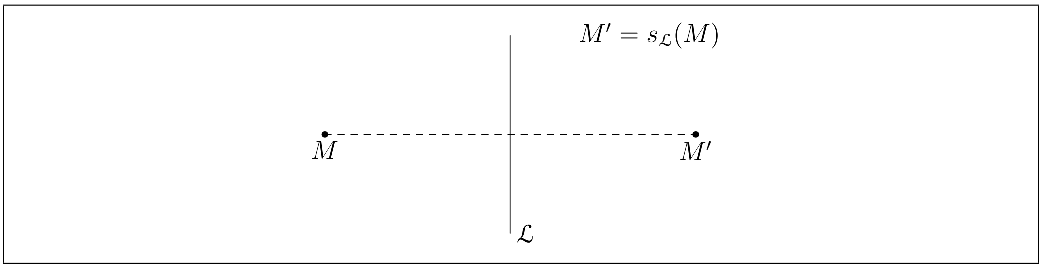
5.1.2 Reflection in a line

DEFINITION 5–2:

Let $\mathcal{L}$ be a line. The *reflection* [rɪˈflekʃn] *in* (or *about* or *through*) *line* $\mathcal{L}$ is the transformation which to each point M in the plane associates the point M' such that $\mathcal{L}$ is the midperpendicular of $[MM']$ if M is not on $\mathcal{L}$ and such that $M' = M$ if M is on $\mathcal{L}$.

The reflection in $\mathcal{L}$ is usually denoted by $s_{\mathcal{L}}$.

Fig. 5–2



PROPOSITION 5–2:
Let A and B be two points in the plane. Let $\mathcal{L}$ be a line and s be the reflection in $\mathcal{L}$. Let $A' = s(A)$ and $B' = s(B)$.

- $A'B' = AB$;
- if (AB) is perpendicular to $\mathcal{L}$, then $(A'B') = (AB)$;
- if (AB) is parallel to $\mathcal{L}$, then $(A'B')$ is also parallel to $\mathcal{L}$.

PROPOSITION 5–3:
Let A and B be two points in the plane. Let $\mathcal{L}$ be a line and s be the reflection in $\mathcal{L}$. Let $A' = s(A)$ and $B' = s(B)$.
Assume[1] [əˈsjuːm] that (AB) is not parallel to $\mathcal{L}$. Let I be their common point. Then I is *invariant* under s i.e. $s(I) = I$. Moreover, I is the only invariant point on (AB).

5.1.3 Reflection in a point

DEFINITION 5–3:

Let O be a point. The *reflection in* (or *about* or *through*) *point* O is the transformation which to each point M in the plane associates the point M' such that O is the midpoint of $[MM']$.

The reflection in O is usually denoted by S_O.

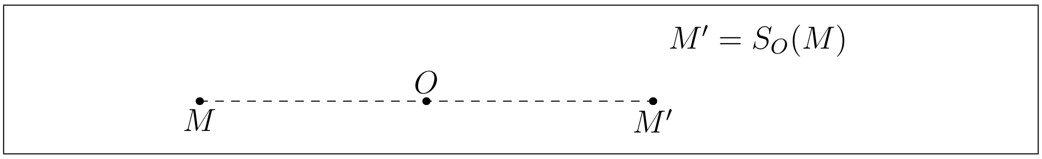


Fig. 5–3

PROPOSITION 5–4:
Let A and B be two points in the plane. Let O be a point and S be the reflection in O. Let $A' = S(A)$ and $B' = S(B)$.

- $\overrightarrow{A'B'} = -\overrightarrow{AB}$ and consequently,
- $A'B' = AB$,
- $(A'B')$ is parallel to (AB);
- the only invariant point under S is O.

[1] To assume: take or accept as being true for the purpose of argument.

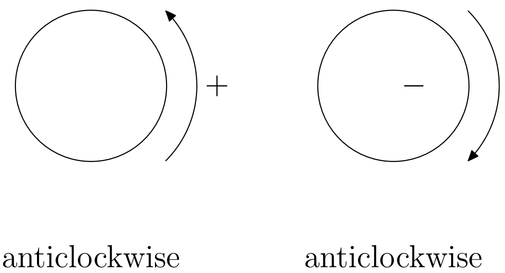
5.2 Orientation of the plane, rotations

5.2.1 Orientation of the plane

There are two directions on a circle: the clockwise and the anticlockwise direction. The anticlockwise direction is chosen to be the *positive direction*. The other one is the *negative direction*.

To give a direction to the plane is to choose the same direction for all the circles in the plane. The plane is then said to be *directed*. The plane is generally assumed to be positively directed.

Fig. 5–4

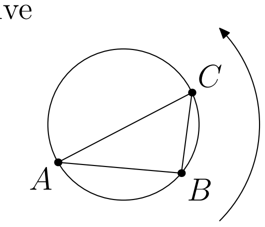


Definition 5–4:

The triangle ABC is said to be *positive* if on the circumcircle of ABC one goes from A to C through B in the positive direction. If ABC is not positive then it is said to be *negative*.

Fig. 5–5

ABC is positive



***Remark**: If ABC is positive then BCA and CAB are also positive whereas CBA, BAC and ACB are negative.*

Definition 5–5:

The triangles ABC and DEF are said to have the *same sense* if they are both positive or both negative.

5.2.2 Rotation

Definition 5–6:

The *rotation of angle α anticlockwise about O* is the transformation which to each point $M \neq O$ in the plane associates the point M' such that:

- if $M \neq O$ then $OM = OM'$, OMM' is positive and $\angle MOM' = \alpha$;
- if $M = O$ then $M' = O$.

***Remark**: The rotation of $\alpha°$ anticlockwise about O is equal to the rotation of $(360 - \alpha)°$ clockwise about O. Therefore, the rotation of $180°$ anticlockwise about O is equal to the rotation of $180°$ clockwise about O and to the reflection in O.*

Proposition 5–5:

Let A and B be two points in the plane. Let O be a point and R be the reflection of $\alpha°$ anticlockwise about O. Let $A' = R(A)$ and $B' = R(B)$. Let C be such that $\overrightarrow{AC} = \overrightarrow{A'B'}$. Then:

- $A'B' = AB$;
- $\angle BAC = \alpha$;
- If the angle of R is not null then O is the only invariant point under R.

Proposition 5–6:

Translations and rotations — and therefore reflections in a point — map a triangle onto a triangle with the same sense. Reflections in a line map a triangle onto a triangle with the opposite sense.

5.3 Isometries

Definition 5–7:

An *isometry*[2] [aɪˈsɒmɪtrɪ] is any transformation from the plane onto itself that preserves distance.

In other words, if h is an isometry, A and B two points in the plane, A' and B' their images under h respectively, then $A'B' = AB$.

[2] from Greek ‘isometria’: equality of measure. The prefix ‘iso’ means ‘equal’ as in ‘isosceles’: having two equal legs. And ‘metria’ is the measure as in ‘geometry’ which is — originally — the science of the measure of Earth.

PROPOSITION 5–7:
Translations, reflections and rotations are isometries.

We can state the following useful properties of isometries:

PROPOSITION 5–8:
Let A, B and C be three points in the plane. Let h be any isometry. Let $A' = h(A)$, $B' = h(B)$ and $C' = h(C)$.

- the image under h of a circle of center A and radius r, is a circle of center A' and radius r;
- the image under h of the segment $[AB]$ is the segment $[A'B']$;
- if I is the midpoint of $[AB]$ then $h(I)$ is the midpoint of $[A'B']$;
- the image under h of the line (AB) is the line $(A'B')$;
- if $\angle ABC$ is right then $\angle A'B'C'$ is right;
- if the lines $\mathcal{L}$ and $\mathcal{L}'$ are parallel then $h(\mathcal{L})$ and $h(\mathcal{L}')$ are parallel;
- $\angle ABC = \angle A'B'C'$.

5.4 Composition

Let h and k be two transformations of the plane. Let M be a point in the plane. We can first apply h to any point M and, in so doing, obtain point M^*. Then we can apply k to point M^* and obtain M'. The transformation which maps M onto M' is said to be the *composite* [ˈkɒmpəzɪt] of h with k. We have composed [kəmˈpəʊzd] those two transformations to obtain a new one. That operation is called the *composition* [ˌkɒmpəˈzɪʃn].

Usually, the composite of h with k — i.e. h followed by k — is denoted by $k \circ h$ (read 'k circle h' or 'k of h'). The first transformation to operate is written on the right hand side for it is there that we write the point to be transformed: $k \circ h(M)$. Here we can write $k \circ h(M) = k(M^*) = M'$.

5.4.1 Composition of two translations

PROPOSITION 5–9:
The composite of the translation by $\overrightarrow{u}$ with the translation by $\overrightarrow{v}$ is the translation by $\overrightarrow{u} + \overrightarrow{v}$.

$$t_{\overrightarrow{v}} \circ t_{\overrightarrow{u}} = t_{\overrightarrow{u}} \circ t_{\overrightarrow{v}} = t_{\overrightarrow{u}+\overrightarrow{v}}$$

The second figure below is a schematic [skɪˈmætɪk] representation of the composition of two translations. The first to operate maps M onto M^* then the second maps M^* onto M'. The short-cut from M to M' represents the composite of the two preceding translations.

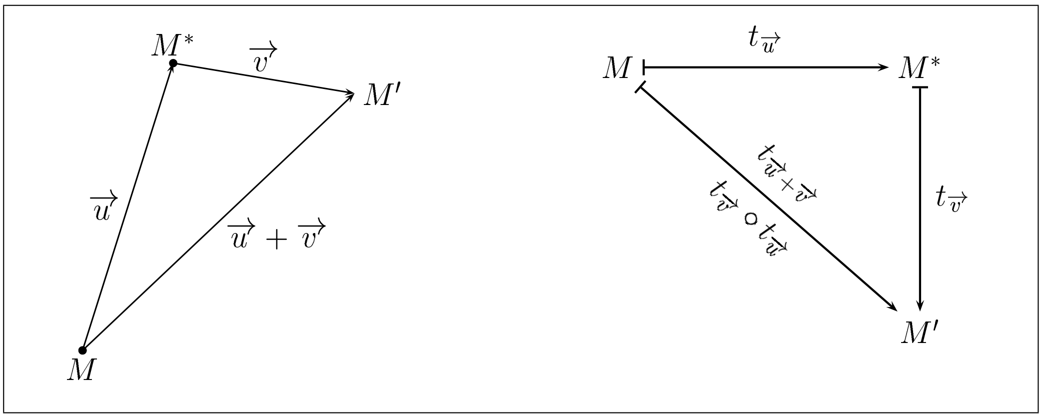


Fig. 5–6

5.4.2 Composition of two reflections in a point

Proposition 5–10:
The composite of the reflection in O with the reflection in O' is the translation by $2\overrightarrow{OO'}$.

$$S_{O'} \circ S_O = t_{2\overrightarrow{OO'}} \quad \text{and} \quad S_O \circ S_{O'} = t_{2\overrightarrow{O'O}}$$

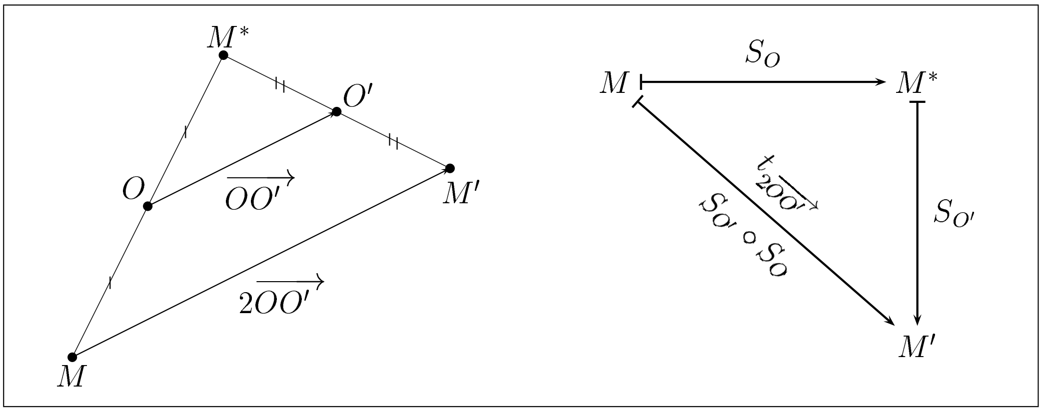


Fig. 5–7

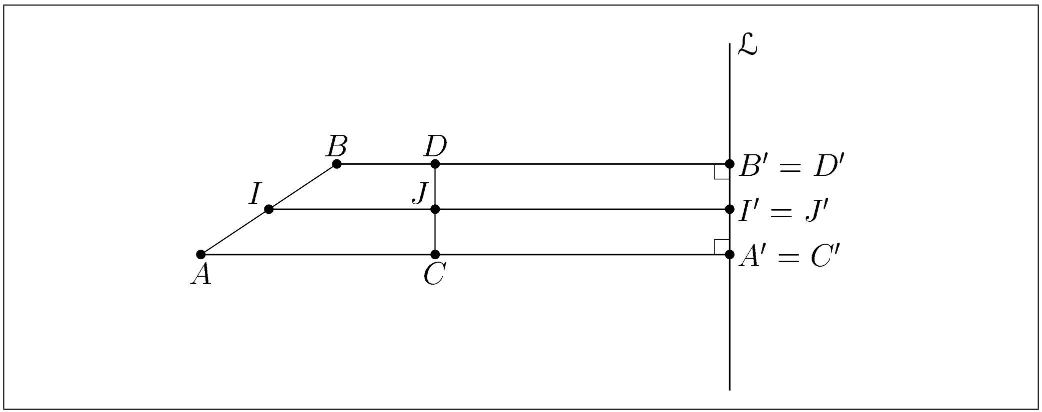
Try your hand at proving that, for any point M, $\overrightarrow{MM'}$ is truly equal to $2\overrightarrow{OO'}$. Use the figure above and the triangle law (see chap.4 prop. 6).

5.5 Projections

Definition 5–8:

The orthogonal projection from the plane to line $\mathcal{L}$ is the mapping which to each point M in the plane associates the point M' such that M' is the intersection point of $\mathcal{L}$ and the perpendicular to $\mathcal{L}$ passing through M.

Fig. 5–8



***Remark**: A projection is not an isometry for it does not map the plane onto itself. Moreover a projection does not preserve length. (See figure above where $AB \neq CD$ and $A'B' = C'D'$.*

Proposition 5–11:

A projection preserves the midpoints i.e. if I is the midpoint of $[AB]$ and if A', B' and I' are the respective image under the projection on $\mathcal{L}$ then I' is the midpoint of $[A'B']$.

Chapter 6

Solid geometry

Solid geometry or *stereometry* [ˌsterɪˈɒmɪtrɪ] deals with the objects of the space such as points, lines, planes and so on.

6.1 Definitions and first properties

6.1.1 Points, lines and planes

PROPOSITION 6–1:
Two points determine a line.

The line passing through the points A and B is denoted by (AB).

PROPOSITION 6–2:
Three non-collinear points determine a plane.

If A, B and C are three non-collinear points, the plane they determine is denoted by (ABC). A is in the plane (ABC). One can write: $A \in (ABC)$.

Points which are in the same plane are said to be *coplanar* [kəʊˈpleɪnəʳ].

PROPOSITION 6–3:
If A and B are in the plane $\mathcal{P}$ then all the points of the line (AB) are in the plane $\mathcal{P}$. The line (AB) is said to be in the plane $\mathcal{P}$.

Instead of 'the line $\mathcal{L}$ is in the plane $\mathcal{P}$', one can say 'the line $\mathcal{L}$ is *included in* the plane $\mathcal{P}$'.

One writes: $\mathcal{L} \subset \mathcal{P}$ to denote that the line $\mathcal{L}$ is in the plane $\mathcal{P}$.

PROPOSITION 6–4:
Two intersecting lines determine a plane. So two intersecting lines are coplanar.
One line and one point not on the line determine a plane.

6.1.2 Parallelism

One uses the sign $\varnothing$ to denote an empty set, for example $\mathcal{L} \cap \mathcal{L}' = \varnothing$ means that lines $\mathcal{L}$ and $\mathcal{L}'$ have no common point.

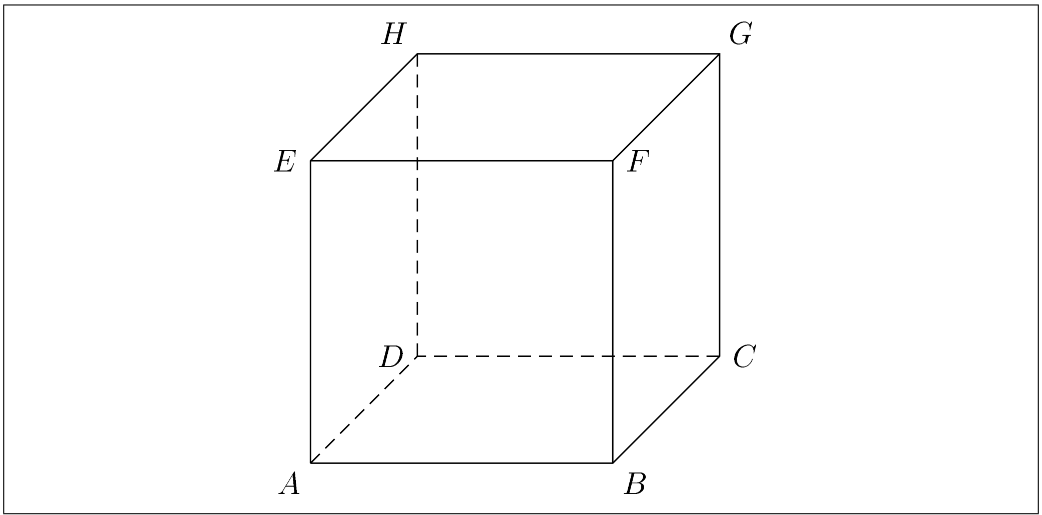
DEFINITION 6–1:

Two lines are *parallel* if and only if they are coplanar and do not intersect.

Two parallel lines which are not identical have no common point. If two parallel lines have a common point then they are identical. So if $\mathcal{L}$ and $\mathcal{L}'$ are parallel or $\mathcal{L} = \mathcal{L}'$ or $\mathcal{L} \cap \mathcal{L}' = \varnothing$. In the space two lines can have no common point without being parallel. That is the case when they are not coplanar then they are said to be *skew* [skjuː].

The following figure is a *cabinet drawing* [ˈkæbɪnɪt ˈdrɔːɪŋ] of the cube $ABCDHEFG$.

Fig. 6–1



***Remark**: (AB) and (CG) do not meet and are not parallel. They are not coplanar. (AC) and (BD) are coplanar but not parallel.*

Pictorial [pɪkˈtɔːrɪəl] *drawing* shows a ‘picture’ of an object in three dimensions, as if one were looking at the object itself. One form of pictorial drawing is *oblique* [əˈbliːk] *drawing*. In oblique drawing, a front view of the object is drawn. From the front view, lines at any angle are drawn. This angle is usually at 45 degrees, although other angles are used. Cabinet drawing is the most common form of oblique drawing. In this form of oblique drawing, the sloping lines are at 45 degrees and the lengths along the sloping lines are drawn to half scale.

PROPOSITION 6–5:

Two parallel lines without a common point determine a plane.

Definition 6–2:

Two planes are *parallel* if and only if they are identical or if they have no common point.

On the figure 6–1, planes (ABC) and (EFG) are parallel.

Proposition 6–6:

If two planes are not parallel they intersect in a line.

In other words, if $\mathcal{P}$ and $\mathcal{P}'$ are not parallel then $\mathcal{P} \cap \mathcal{P}'$ is a line.

On the figure 6–1, planes (ABC) and (FEA) intersect in the line (AB).

Proposition 6–7:

Let planes $\mathcal{P}$ and $\mathcal{Q}$ be parallel. If a plane $\mathcal{R}$ intersects $\mathcal{P}$ then it intersects $\mathcal{Q}$. Let then d be $\mathcal{P} \cap \mathcal{R}$ and δ be $\mathcal{Q} \cap \mathcal{R}$, d and δ are parallel.

Definition 6–3:

The line $\mathcal{L}$ is *parallel* to the plane $\mathcal{P}$ if $\mathcal{L}$ is in $\mathcal{P}$ or if $\mathcal{L}$ and $\mathcal{P}$ have no common point.

Beware! On the figure 6–1, lines (EF) and (EG) are both parallel to the plane (ABC) *but* they *do* intersect. Parallelism is a relation which is innocuous when dealing with lines only or with planes only but which is rather dangerous when dealing with lines and planes at the same time.

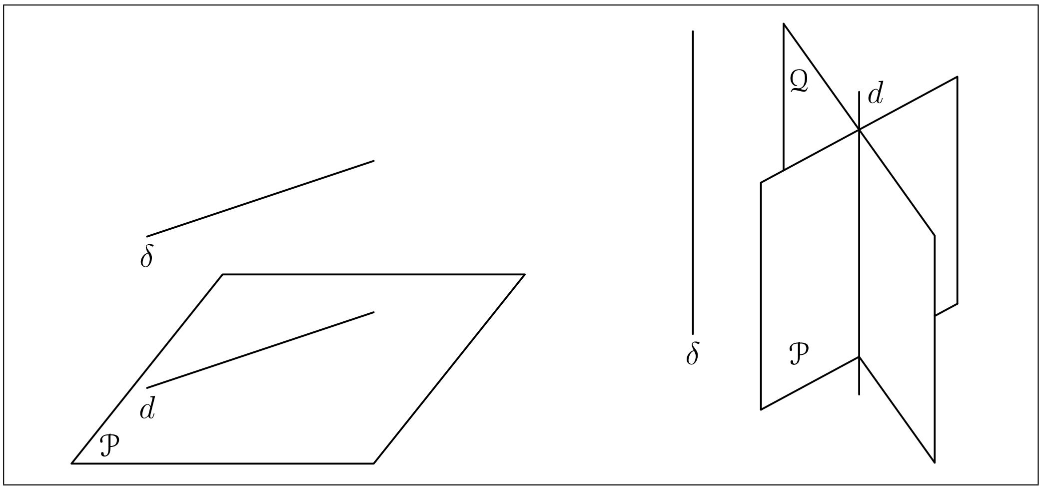


Fig. 6–2

We can now state these usefull propositions:

Proposition 6–8:

If the line δ is parallel to the line d and if the line d is in the plane $\mathcal{P}$ then the line δ is parallel to the plane $\mathcal{P}$. (See figure 6–2 left)

Proposition 6–9:
If the line δ is parallel to the plane $\mathcal{P}$ and to the plane $\mathcal{Q}$ and if the two planes intersect in the line d then the line δ is parallel to the line d. (See figure 6–2 right).

6.1.3 Orthogonality

Definition 6–4:

Two lines of space are said to be *perpendicular* [ˌpɜːpenˈdɪkjʊləʳ] iff they are coplanar and perpendicular in their common plane.

Definition 6–5:

Two lines d and δ are said to be *orthogonal* [ɔːˈθɒg(ə)n(ə)l] iff there exists a line which is parallel to d and perpendicular to δ.

So orthogonal lines are not necessarily coplanar and do not always intersect. For example, (AB) and (AD) are perpendicular whereas (AB) and (DH) are orthogonal and do not intersect. (See figure 1)

If the lines are not coplanar some strange events can happen: (AB) is perpendicular to (AD) which is perpendicular to (DH) but (AB) and (DH) are *not* parallel. The theorem which states that *if two lines are perpendicular to a third line then they are parallel*, is true only if the three lines are coplanar.

Definition 6–6:

The line d is said to be perpendicular to the plane $\mathcal{P}$ iff d is orthogonal to two intersecting lines of $\mathcal{P}$.

Proposition 6–10:
If the line d is perpendicular to the plane $\mathcal{P}$ then d intersects $\mathcal{P}$ moreover d is orthogonal to each line of $\mathcal{P}$.

Proposition 6–11:

- If two lines are perpendicular to the same plane then they are parallel.
- If a line is perpendicular to two planes then the planes are parallel.

Definition 6–7:

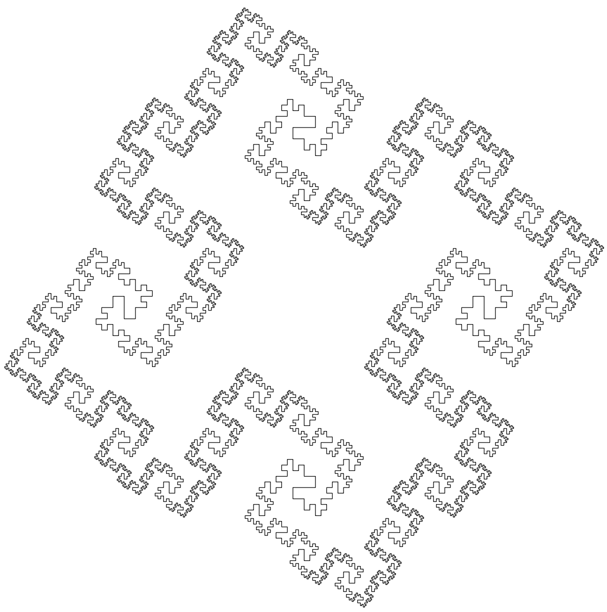
Let A and B be two distinct points. The plane $\mathcal{P}$ is said to be the *perpendicular bisector* [baɪˈsektəʳ] of $[AB]$ iff $\mathcal{P}$ passes through the midpoint of $[AB]$ and (AB) is perpendicular to $\mathcal{P}$.

PROPOSITION 6–12:

The perpendicular bisector of $[AB]$ is the set of all the points which are equidistant [ˌiːkwɪˈdɪstənt] from A and B.

DEFINITION 6–8:

The plane $\mathcal{Q}$ is said to be perpendicular to the plane $\mathcal{P}$ iff $\mathcal{Q}$ contains a line which is perpendicular to $\mathcal{P}$.



Chapter 7

Algebra

7.1 Introduction

Algebra [ˈældʒɪbrə], like *arithmetic* [əˈrɪθmetɪk], deals with numbers. Algebra is an extension of arithmetic. Both subjects employ the fundamental operations. In arithmetic you use numbers whose values are known; you operate with these and obtain definite numerical [njuːˈmerɪkl] results. Whereas in algebra, while you may use definite numbers on accasions, you are, in the main, concerned with general expressions and general results, in which letters or other symbols represent numbers not named or specified.

There is a rule to determine the area of a rectangle which might be expressed in the form: '*the area of a rectangle in square meters is equal to the product of the length in metres by the breadth in metres*'. This rule is shortened by employing letters as symbols, to represent the quantities. Thus:

Let l represent the 'lenght in metres', b represent the 'breadth in metres' and A represent 'the area in square metres'. With those symbols you can now write the above rule in the form: $A = l \times b$.

This shortened form shows the rule for finding the area of any rectangle; it is a general rule, and is called a *formula* [ˈfɔːmjʊlə]. A is said to be the *subject* of the formula. In that formula A is expressed in terms of l and b. You can change the subject of the formula, for example $b = A/l$.

If you wish to find the numerical value of an algebraic expression for definite numerical values of the letters in it, you should *substitute* [ˈsʌbstɪtjuːt] the numerical values for the letters. So, when $l = 3$ and $b = 2$, substituting the given values leads to $A = 6$.

It frequently happens that an expression, or part of an expression, is to be operated as a whole. For example, suppose that you wish to write in algebraic [ˌældʒɪˈbreɪɪk] symbols 'Twice the sum of a and b'. Brackets are used to enclose the part which is to be operated on as a whole — namely, $a + b$. You write $2\,(a + b)$.

The expression $3a$ denotes a multiple of a and the number 3, which indicates the multiple, is called the *coefficient* [ˌkəʊɪˈfɪʃnt] of a. In $3ab$, 3 is the coefficient of ab, $3a$ is the coefficient of b and $3b$ is the coefficient of a.

In a expression, terms which involve the same letter, and differ only in the coefficients of this letter, are called *like terms*. Thus in the expression $3a + 5b - 2a + 4b$, $3a$ and $2a$ are like terms.

7.2 Equations

An *equation* [ɪˈkweɪʒn] is an expression that represents the *equality* [ɪˈkwɒlətɪ] of two expressions involving constants [ˈkɒnst(ə)nt] or variables [ˈveərɪəbl]. For example, the equation $A = \pi r^2$ *equates* [ɪˈkweɪt] the area A of a circle *to* the product πr^2.

Let x be an *unknown* number. To *solve* [sɒlv] the equation $5x = 40$ is to find the value(s) of x which satisfy(fies) the equality. Those values are called the solutions [səˈluːʃn] of the equation.

Proposition 7–1:

- If the same number is added to, or subtracted from, both sides of an equation, the two sides will again be equal.
- If both sides of an equation are multiplied or divided by the same number, the two sides of the new equation will be equal.

7.2.1 Example

Solve the equation: $6x - 5 = 2x + 9$.

The general plan adopted is to collect the terms involving the unknown on the left (hand) side, and the other terms on the right. Transferring the x terms from the right side you get:

$$6x - 2x - 5 = 9.$$

Transferring the -5,

$$\begin{aligned} 6x - 2x &= 9 + 5 \\ \therefore \qquad 4x &= 14 \\ \text{and} \qquad x &= \frac{7}{2}. \end{aligned}$$

The solution is $\frac{7}{2}$. The *solution set* is $\left\{\frac{7}{2}\right\}$ i.e. the set containing $\frac{7}{2}$ as its only element.

***Remark**: The sign ∴ denotes 'therefore'.*

7.3 Simultaneous equations

Simultaneous [ˌsɪməlˈteɪnɪəs] *equations* are where we have a pair (or more) of equations that both (or all) need solving at the same time — hence simultaneous. There are two main methods: the *elimination* [ɪˌlɪmɪˈneɪʃn] method and the *substitution* [ˌsʌbstɪˈtjuːʃn] method.

7.3.1 Elimination method

The technique is initially to eliminate one variable to find a solution to the other one, and then substitute the found variable to complete the solution.

Example Solve the simultaneous equations

$$4x + y = 14 \qquad (1)$$
$$2x + y = 8 \qquad (2)$$

To eliminate one of the variables, subtract the equations, botton from top, which will give $2x = 6$, so $x = 3$.

Now, we substitute this answer in the simplest equation, (2) above, to give $6+y = 8$, so $y = 2$. Therefore the solution of the simultaneous equations is $x = 3$ and $y = 2$. The solution set is $\{(3\,;\,2)\}$.

Note that you should always check the solution and see that it works with the other equation.

7.3.2 Substitution method

This is an alternative method for solving simultaneous equations.

Example Solve the simultaneous equations

$$4x - 2y = 11 \qquad (1)$$
$$3x + y = 12 \qquad (2)$$

Then from equation (2), $y = 12 - 3x$. Now substitute this into equation (1) to give

$$\begin{aligned} 4x - 2\,(12 - 3x) &= 11 \\ 4x - 24 + 6x &= 11 \\ 10x &= 35 \\ x &= 3.5 \end{aligned}$$

and we are where we arrived at before. We now need to substitute $x = 3.5$ into one of the equations to complete the solution. So $y = 12 - 3 \times 3.5 = 12 - 10.5 = 1.5$ and the solution set is $\{(\,3.5\,;\,2.5\,)\}$.

7.4 Inequalities

Inequations are often referred to as *inequalities* also. They are solved as equations, except that when you have to multiply or divide *both* sides by a negative number the inequality sign turns round. For example, if we have $-2 < 5$, then $2 > -5$.

Example Find the range of value for which $5x + 3 > 6(x - 2)$.

You have to expand the brackets first to give $5x + 3 > 6x - 12$. Then transferring the x term from the right hand side and the 3 from the left hand side, you obtain $5x - 6x > -12 - 3$ and so $-x > -15$. You have to multiply both sides by -1 and so $x < 15$. The solution set is the interval $]-\infty\,;\,15\,[$.

Chapter 8

Statistics

There are three kinds of lies: lies, damned lies and statistics.
Attributed to Benjamin DISRAELI[1]
Mark TWAIN (1835–1910), American writer,
in *Autobiography* (1924)[2]

Statistics : [stəˈtɪstɪks] **1** (*usually treated as singular*) the science of collecting and analysing numerical [njuːˈmerɪkl] data, especially in or for large quantities, and usually inferring proportions in a whole from proportions in a representative sample. **2** any systematic collection or presentation of such facts.

8.1 Introduction

Every day we make decisions that may be personal, business related, or of some other kind. Usually these decisions are made under conditions of uncertainty. Many times, the situations or problems we face in the real world have no precise or definite solution.

Statistical [stəˈtɪstɪkl] methods help us to make scientific and intelligent decisions in such situations. Decisions made by using statistical methods are called *educated guesses*. Decisions made without using statistical (or scientific) methods are pure guesses and, hence, may prove to be unreliable.

Like almost all fields of study, statistics has two aspects: *theoretical* [θɪə-

[1]Benjamin DISRAELI [dɪzˈreɪlɪ] (1804–1881) 1st Earl of Beaconsfield [ˈbiːkənzfiːld], British Conservative politician and novelist; Prime Minister, 1868, 1874–1880.

[2]'Mark Twain' is the pseudonym [ˈsjuːdənɪm] of Samuel Langhorne CLEMENS.

ˈretɪkl] and *applied.* Theoretical or mathematical statistics deals with the development, derivation, and proof of statistical theorems, formulas, rules, and laws. Applied statistics involves the applications of those theorems, formulas, rules, and laws to solve real-world problems.

Broadly speaking, applied statistics can be divided into two areas: *descriptive* [dɪˈskrɪptɪv] statistics and *inferential* [ˌɪnfəˈrenʃl] statistics. Descriptive statistics consists of methods for organising, displaying, and describing data by using tables, graphs, and summary [ˈsʌmərɪ] measures.

Suppose we have information on the test scores of students enrolled in a statistics class. In statistical terminology, the whole set of numbers that represents the scores of students is called a *data*[3] [ˈdeɪtə] *set*, the name of each student is called an *element* [ˈelɪmənt] or *individual* [ˌɪndɪˈvɪdʒʊəl], and the score of each student is called an *observation* [ˌɒbzəˈveɪʃn].

A data set in its original form is usually very large. Consequently, such data set is not very helpful in drawing conclusions or making decisions. It is easier to draw conclusions from summary tables and diagrams [ˈdaɪəgræm] than from the original version of a data set. Therefore, we reduce data to a manageable size by constructing tables, drawing graphs, or calculating summary measures such as averages [ˈæv(ə)rɪdʒ].

In statistics, the collection of all elements of interest is called a *population* or *universe.* The selection of a few elements from this population is called a *sample* [ˈsɑːmpl].

A major portion of statistics deals with making decisions, inferences, predictions, and forecasts about populations based on results obtained from samples. The area of statistics that deals with such decision-making procedures is referred to as inferential statistics. This branch of statistics is also called inductive [ɪnˈdʌktɪv] reasoning [ˈriːz(ə)nɪŋ] or inductive statistics.

A population consists of all elements — individuals, items [ˈaɪtəm], or objects — whose characteristics are being studied. The population that is being studied is also called the target [ˈtɑːgɪt] population.

The collection of information from the elements of a population or a sample is called a *survey* [ˈsɜːveɪ]. A survey that includes every element of the target population is called a *census* [ˈsensəs]. Often the size of the target population is large. Hence, in practice, a census is rarely taken because it is very expensive and time consuming. In many cases, it is even impossible to identify each element of the target population. Usually, to conduct a survey, we select a sample and collect the required information from the elements included in that sample. We then make decisions based on this sample information. Such a survey is called a *sample survey.*

[3] *plural of* datum [ˈdeɪtəm].

The purpose of conducting a sample survey is to make decisions about the corresponding population. It is important that the results obtained from a sample survey closely match the results that we would obtain by conducting a census. Otherwise, any decision based on a sample survey will not apply to the corresponding population.

A sample that represents the characteristics of the population as closely as possible is called a *representative* [ˌreprɪˈzentətɪv] *sample*.

A sample may be *random* [ˈrændəm] or non-random. In a *random sample*, each element of the population has some chance of being included in the sample. However, in a non-random sample this may not be the case. If the chance of being selected is the same for each element of the population, the sample is called a *simple random sample*.

A sample may be selected with or without *replacement*. In *sampling with replacement*, each time we select an element from the population, we put it back in the population before we select the next one. Thus, in sampling with replacement, the population contains the same number of items each time a selection is made. Consequently, we may select the same item more than once in such a sample.

Sampling without replacement occurs when the selected element is not replaced in the population. In this case, each time we select an item, the size of the population is reduced by one element. Thus, we cannot select the same item more than once. Most of the times, samples taken in statistics are without replacement.[14]

8.1.1 Basic terms

An *element* or *member* of a sample or population is a specific subject or object (for example, a person, firm, item, state, or country) about which the information is collected. A *variable* is a characteristic under study that assumes different values for different elements. The value of a variable for an element is called an *observation* or *measurement*. A *data set* is a collection of observations on one or more variables.

8.1.2 Types of variables

A variable that can be measured numerically [njuːˈmerɪk(ə)lɪ] is called a *quantitative* [ˈkwɒntɪtətɪv] variable. The data collected on a quantitative variable are called quantitative data.

Incomes, heights, gross sales, prices of homes, number of cars owned, and accidents are examples of quantitative variables since each of them can be expressed numerically. Such quantitative variables can be classified as either *discrete* [dɪˈskriːt] variables or *continuous* [kənˈtɪnjʊəs] variables.

The values that a certain quantitative variable can assume may be countable or not. For example, we can count the number of cars owned by a family but we cannot count the income of the family. The variable the values of which are countable is called a discrete variable. There are no possible intermediate values between consecutive values of a discrete variable.

A continuous variable can assume any numerical value over a certain interval or intervals.

The time taken to complete an examination is an example of a continuous variable because it can assume any value, let us say, between 30 and 60 minutes. The time taken may be 42.6 minutes, 42.67 minutes, or 42.674 minutes. (Theoretically, we can measure time as precisely as we want.) Similarly, the height of a person can be measured to the tenth of an inch or to the hundredth of an inch. However, neither time nor height can be counted in a discrete fashion. Note that any variable that involves money is considered a continuous variable.

Variables that cannot be measured numerically but can be divided into different categories [ˈkætəgərɪ] are called *categorical* [ˌkætəˈgɒrɪkl] or *qualitative* [ˈkwɒlɪtətɪv] variables. The data collected on such variables are called qualitative data.

Based on the time over which they are collected, data can be classified as either *cross-section* or *time-series* data. Data collected on different elements at the same point of time or for the same period of time are called cross-section data. The information on incomes of 100 families for the year 1994 is an example of cross-section data.

Data collected on the same element for the same variable at different points in time or for different periods of time are called time-series data. Information on GB exports for the years 1975 to 1993 is an example of time-series data.

When data are collected, the information obtained from each member of a population or sample is recorded in the sequence in which it becomes available. This sequence of data recording is random and unranked. Such data are called *raw* [rɔː] *data*.

8.2 Organising and graphing qualitative data

8.2.1 Frequency distributions

A sample of 100 students enrolled at a university were asked what they intended to do after graduation. Forty-four said they wanted to work for private companies; 16 said they wanted to work for the federal government; 23 wanted to work for state or local governments; and 17 intended to start their own businesses.

The table 1 lists the type of employment and the number of students who intend to engage in each type of employment. In this table, the variable is the type of employment, which is a qualitative variable. The categories listed in the first column of the table are mutually exclusive. In other words, each of the 100 students belongs to one and only one of these categories.

The number of students who belong to a certain category is called the *frequency* [ˈfriːkwənsɪ] of this category. A *frequency distribution* [ˌdɪstrɪˈbjuːʃn] exhibits how the frequencies are distributed over various categories. The next table is called a *frequency distribution table* or simply a *frequency table*.

Table 1

Type of employement	Number of students
Private companies	44
Federal government	16
State or local government	23
Own business	17
	Sum = 100

Definition 8–1:

A *frequency distribution* for qualitative data lists all categories and the number of elements that belong to each of the categories.

8.2.2 Relative frequency and percentage distributions

Definition 8–2:

The *relative frequency* of a category is obtained by dividing the frequency of that category by the sum of all frequencies.

Thus, the relative frequency shows what fractional part or proportion of the total frequency belongs to the corresponding category. A relative frequency distribution lists the relative frequencies for all categories.

DEFINITION 8–3:

The *percentage* for a category is obtained by multiplying the relative frequency of that category by 100. A percentage distribution lists the percentages for all categories.

A sample was taken of 25 high school seniors who were planning to go to college. Each of the students was asked which of the following majors he or she intended to choose: business, economics, management information systems (MIS), behavioral sciences (BS), other. The responses are given in the table below.

Table 2

Major	Frequency (f)	Relative Frequency	Percentage
Business	6	.24	24
Economics	3	.12	12
MIS	6	.24	24
BS	2	.08	8
Others	8	.32	32
	sum = 25	sum = 1.00	sum = 100

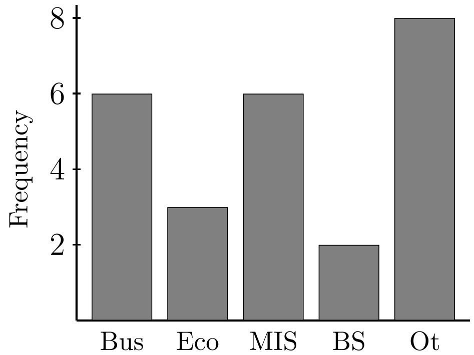
8.2.3 Graphical representation of qualitative data

All of us have heard the saying 'a picture is worth a thousand words'. A graphic display can reveal at a glance the main characteristics of a data set. The *bar graph* [ˈbɑːˌgrɑːf] and the *pie chart* [ˈpaɪˌtʃɑːt] are two types of graphs used to display qualitative data.

Bar graph

To construct a bar graph (also called a *bar chart*), we mark the various categories on the horizontal axis as in figure 8–1. Note that all categories are represented by intervals of the same width.

We mark the frequencies on the vertical axis. Then we draw one bar for each category such that the height of the bar represents the frequency of the corresponding category. We leave a small gap between adjacent bars.



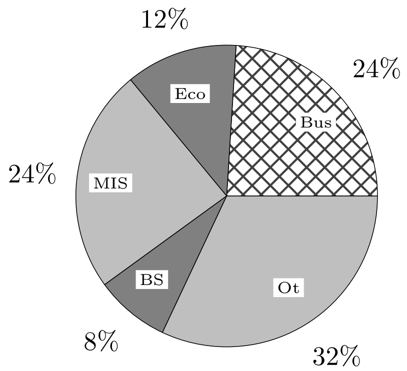
Bar chart for the frequency distribution of Table 2

Fig. 8–1

The bar graphs for relative frequency and percentage distributions can be drawn simply by marking the relative frequencies or percentages, instead of the frequencies, on the vertical axis.

Pie chart

A pie chart is more commonly used to display percentages, although it can be used to display frequencies or relative frequencies. The whole pie (or circle) represents the total sample or population. The pie is divided into different portions that represent the percentages of the population or sample belonging to different categories.



Pie chart for the frequency distribution of Table 2

Fig. 8–2

8.3 Organising and graphing quantitative data

The next table gives the heights (in inches) of a random sample of 30 National Basketball Association players. The first column of this table lists the classes, which represent the quantitative variable, the height.

For quantitative data, an interval that includes all the values that fall within two numbers, the *lower* and *upper limits* (or *class boundaries* [ˈbaʊn-d(ə)rɪ]) is called a *class*. Note that the classes always represent a variable. As we can observe, the classes are non-overlapping; that is, each value belongs to one and only one class. We have to understand that 72–74 stands for $[\,72\,;\,75\,[$ as the lower limit of the next class is 75. So the boundaries of the first class are 72 and **75**.

The third column in the table lists the number of players who have their heights within each class. For example, three players have an height of 72 inches to less than 75 inches. The numbers listed in the third column of this table are called the *frequencies*. The sum of all the frequencies is denoted by Σf. The frequencies are denoted by f.

Table 3

Height (in inches)	class midpoint	Frequency (f)
72–74	73.5	3
75–77	76.5	5
78–80	79.5	7
81–83	82.5	10
84–86	85.5	5
		$\Sigma f = 30$

An *array* is an arrangement of raw numerical data in ascending or descending order of magnitude. When summarising large masses of raw quantitative data, it is often useful to distribute the data into *classes* and to determine the number of individuals belonging to each class, called the *class frequency*. A tabular arrangement of data by classes together with the corresponding class frequency is called a *frequency distribution*, or *frequency table*. Data organised and summarised in that way are often called *grouped data*. The data presented in table 3 are an illustration of a frequency distribution table for quantitative data.

A symbol defining a class, such as 72–74 or $[\,72\,;\,75\,[$, is called a *class interval*. The *class width* (or *class size*) is equal to the difference between the two class boundaries. Here we have $75 - 72 = 3$ so the width of the first class is 3. We can observe that all classes in table 3 have the same size.

The *class mark* is the midpoint of the class interval. It is also called the *class midpoint*. For purposes of further mathematical analysis, all observations belonging to a given class interval are assumed to coincide with the class mark. The mark is obtained by halving the sum of the two boundaries of a class. So the midpoint of the first class is $\frac{75+72}{2} = 73.5$.

8.3.1 Constructing frequency distribution table

While constructing a frequency distribution table, we have to make the following three decisions:

- Number of classes Usually the number of classes for a frequency distribution table varies from 5 to 20, depending mainly on the number of observations in the data set. It is preferable to have more classes as the size of a data set increases.

- Class width Although it is not uncommon to have classes of different sizes, most of the time, it is preferable to have the same width for all classes. To determine the class width when all classes are of the same size, first find the difference between the largest and the smallest values in the data. Then the approximate width of a class is obtained by dividing that difference by the number of desired classes.

- Lower limit of the first class or the starting point Any convenient number, which is equal to or less than the smallest value in the data set, can be used as the lower limit of the first class.

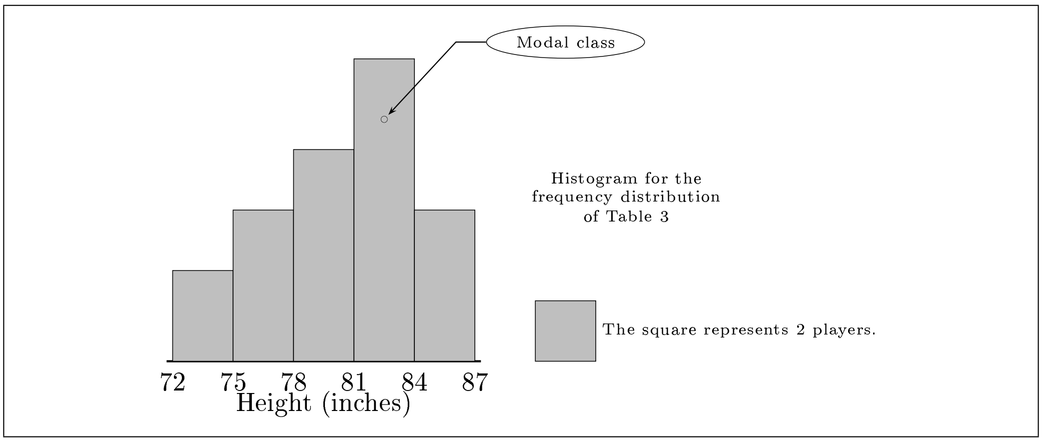
8.3.2 Graphing grouped data

Grouped data can be displayed by using a *histogram* [ˈhɪstəˌgræm] or a *polygon* [ˈpɒlɪgən].

Histograms

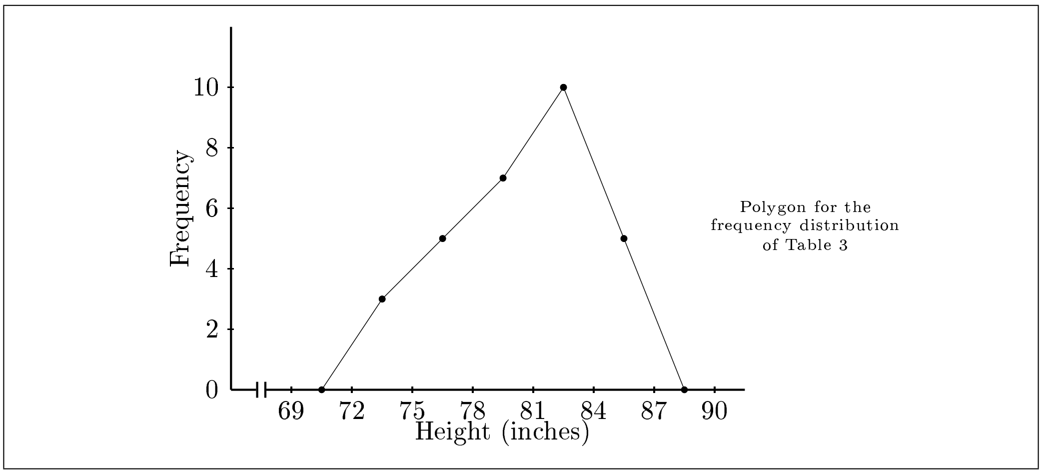
To draw a histogram, we first mark classes on the horizontal axis. Class boundaries are used to mark classes on the horizontal axis. Next, we draw a bar for each class so that its area represents the frequency of this class. The bars in a histogram are drawn adjacent to each other without leaving any gap between them.

Fig. 8–3



Polygons

Fig. 8–4



A polygon is another device that can be used to present quantitative data in graphic form. To draw a frequency polygon, we first mark a dot above the midpoint of each class at a height equal to the frequency of that class. Next, we mark two more classes, one at each end, and mark their midpoints. These two classes have zero frequencies. In the last step, we join the adjacent dots with straight lines. The resulting line graph is called a frequency polygon or simply a polygon.

A polygon with relative frequencies (respectively: percentages) marked on the vertical axis is called a relative frequency (respectively: percentage) polygon.

The symbol '┤├' used in the horizontal axis of figure 8–4 represents a break, called the *truncation* [trʌŋˈkeɪʃn], in the axis. It indicates that the entire axis is not shown in this figure. As can be noticed, the zero to 69 portion of the axis has been omitted.

Warning

Describing data using graphs helps to give us insights into the main characteristics of the data. However, graphs, unfortunately, can also be used, intentionally or unintentionally, to distort the facts and to deceive the reader. Following are two ways of manipulate graphs to convey a particular opinion or impression.

- Changing the scale either on one or on both axes, that is, shortening or stretching one or both axes.

- Truncating the frequency axis, that is, starting the frequency axis at a number greater than zero.

When interpreting a graph, we should be very cautious. We should observe carefully whether the frequency axis has been truncated or whether any axis has been unnecessarily shortened or stretched.

8.3.3 Cumulative frequency distributions

Consider the example about the heights of 30 NBA players. Suppose we want to know how many players are 80 inches tall or shorter. Such a question can be answered using a *cumulative* [ˈkjuːmjʊlətɪv] *frequency distribution*. Each class in a cumulative frequency distribution table gives the total number of values that fall below a certain value. A cumulative frequency distribution is constructed for quantitative data only.

Table 4

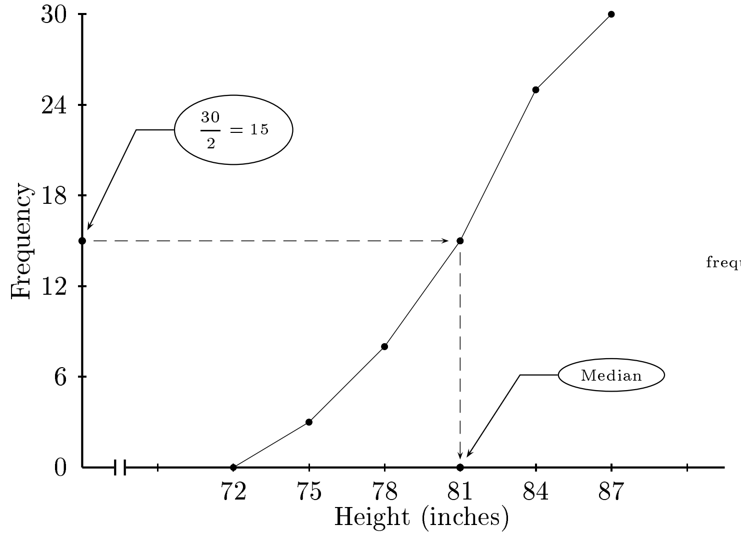
Height (in inches)	Cumulative Frequency
72–74	3
75–77	8
78–80	15
81–83	25
84–86	30

The *cumulative relative frequencies* are obtained by dividing the cumulative frequencies by the total number of observations in the data set. The *cumulative percentages* are obtained by multiplying the cumulative relative frequencies by 100.

Ogives

When plotted on a diagram, the cumulative frequencies give a curve that is called an *ogive* [ˈəʊdʒaɪv]. To draw the ogive, the variable (height) is marked on the horizontal axis and the cumulative frequencies on the vertical axis. Then, the dots are marked above the upper boundaries of various classes at the heights equal to the corresponding cumulative frequencies. The ogive is obtained by joining consecutive points with straight-line segments. Note that the ogive starts at the lower boundary of the first class and ends at the upper boundary of the last class.

Fig. 8–5



Ogive for the frequency distribution of Table 3

8.4 Measures of central tendency

We often represent a data set by numerical summary measures, usually called the typical values. A measure of central tendency or average gives the centre of a histogram or a frequency distribution curve. There are three main types of average — the mean, the mode, and the median; and the first type may be subdivided into arithmetic mean, geometric mean, and harmonic mean. These are all different types of representative value and, being recognised as such, the noun average is rarely used in statistical work when referring to actual values.

8.4.1 Mean

The simplest form of *mean* [miːn] is the arithmetic [ˌærɪθˈmetɪk] mean. It is the most frequently used *measure of central tendency* [ˈtendənsɪ]. It is the easiest to calculate. One just adds up all the values, divides by the total number of values, and the result is the arithmetic mean.

To calculate the mean for grouped data, first find the midpoint of each class and then multiply the midpoints by the frequencies of the corresponding classes. The sum of those products gives an approximation for the sum of all values. To find the value of the mean, divide the sum by the total number of observations in the data.

8.4.2 Outliers or extreme values

Values that are very small or very large relative to the majority of the values in a data set are called outliers [ˈaʊtˌlaɪəʳ] or extreme values.

A major shortcoming of the mean as a measure of central tendency is that it is very sensitive to outliers.

8.4.3 Median

The *median* [ˈmiːdɪən] is the value of the middle term in a data set that has been ranked in increasing order.

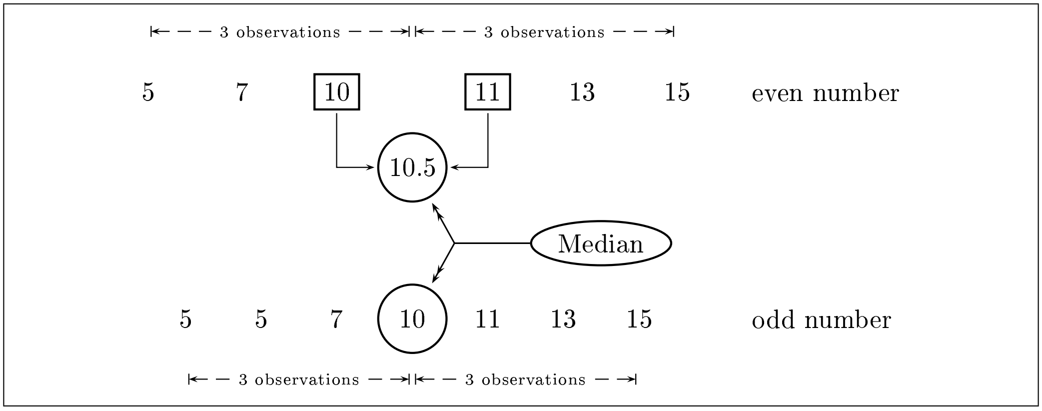
To determine the median for ungrouped data:

- if the number of observations n in a data set is odd, then the median is given by the value of the middle term in the ranked data that is the $(n + 1/2)$th value;

- if the number n is even, then the median is given by the average of the values of the two middle terms the $(n/2)$th and $(n/2 + 1)$th.

To determine the median of grouped data we use the ogive. The median is the abscissa of the point the ordinate of which is half of the total number of observations.

Fig. 8–6



8.4.4 Mode

Mode is a French word that means fashion — an item that is most popular or common. In statistics, the *mode* [məʊd] represents the most common value in a data set; it is the value that occurs with the highest frequency in a data set.

8.4.5 Measure of dispersion

The *range* [reɪndʒ] is the simplest *measure of dispersion* [dɪˈspɜːʃn]. It is the difference between the largest and the smallest values in a data set.

8.5 Samples

The basic idea of sampling is probably almost as old as mankind and, for all we know, possibly preceded it since some lower animals appear to have the knack of tasting a proportion of the food offered to them and of rejecting the whole meal on the strength of the one portion tasted! Sampling is based on choice and selectiveness. Early man had but few possessions and he did not need to count them nor to compare one with another. But as soon as the barter system began to assume larger scale proportions, the two parties to the exchange, say, of coconuts and local barley-wine, would not have tasted every coconut and every drop of wine. Instead they would have tested a few units of each commodity and would have assumed that the rest of the units were up to the standard of the sample units which they had selected. The purchaser of the wine, for instance, could not have tested all the wine without becoming highly intoxicated and probably insensible to the whole transaction. He might, perhaps, have tasted a little from each skin of wine, but in doing this he would still have been sampling on the theory that the remainder of the wine in each skin was exactly the same as the small amounts tasted, although his judgement might have suffered as he proceeded.

It is equally impossible today to inspect every item separately. Electric light bulbs, for instance, are tested for the longevity of useful life by lighting them until the elements break. If every bulb was tested in this way, there would not be any bulbs left to put into the lampholders at home. One hundred per cent inspection like this destroys the article, and some form of sampling is necessary to avoid this wholesale destruction while at the same time producing evidence that the bulk from which the sample is drawn is being maintained at a specified level of quality. Other populations are so large that it would be physically impossible to gather data in respect of every member. Correct sampling methods therefore make it possible to gather information which would otherwise be unobtainable.[24]

We are often interested in knowing the proportion of a population that possesses a certain characteristics, for example, the proportion of Englishmen who own an electric kettle. The *population proportion*, denoted by p, is the ratio of the number of elements in a population with a specific characteristic (here electric kettle owners) to the total number of elements in the population. As the population is too large to be studied directly, we may use a sample. Let n be the *sample size*, that is the number of elements in the sample. The *sample proportion*, denoted by $\hat{p}$ (read p hat) is the ratio of number of elements in the sample with a specific characteristic to the total number of elements in the sample. The value of $\hat{p}$ is an *estimate* of p.

For different samples may lead to different values of $\hat{p}$, we say that $\hat{p}$ is subject to variability. Nonetheless, if the sample fulfils certain conditions, $\hat{p}$ is a good approximation of p. More precisely, if the sample is a simple random sample and if its size is great enough (not less than 25) then:

Proposition 8–1:

More than 93% of all the possible samples lead to a value of $\hat{p}$ which is between $p - \dfrac{1}{\sqrt{n}}$ and $p + \dfrac{1}{\sqrt{n}}$.

Practically a random sample is considered as simple if the population is large enough or if the sampling is made without replacement.

Some consequences follow immediately:

- The bigger the sample, the more precise the estimate.
- $p - \dfrac{1}{\sqrt{n}} \leq \hat{p} \leq p + \dfrac{1}{\sqrt{n}} \Longleftrightarrow \hat{p} - \dfrac{1}{\sqrt{n}} \leq p \leq \hat{p} + \dfrac{1}{\sqrt{n}}$.

The second consequence is of pratical interest for when we perform a sampling the population proportion p is not known. What we know, after the survey, is the value of $\hat{p}$. We can then be 93% certain that the true value of p is somewhere in $\left[\hat{p} - \dfrac{1}{\sqrt{n}}\ ;\ \hat{p} + \dfrac{1}{\sqrt{n}}\right]$ which is known as the 93% *confidence interval*, 93% being the *confidence level*.

Chapter 9

Functions

Still glides the Stream, and shall for ever glide;
The Form remains, the Function never dies.
William WORDSWORTH (1770–1850), English poet,
in *The River Duddon* (1820)

9.1 Definitions

DEFINITION 9–1:

A *function* [fʌŋkʃn] from $\mathcal{D}$ to $\mathbb{R}$ is a rule that maps each real of a certain subset $\mathcal{D}$ of $\mathbb{R}$ onto one and only one real.

Let f be a function from $\mathcal{D}$ to $\mathbb{R}$. Let x be a real belonging to $\mathcal{D}$. The unique real associated to x by f is denoted by $f(x)$ 'f of x' and called the *image* [ˈɪmɪdʒ] of x under f. In such a case, x is called the *independant variable* [ˌɪndɪˈpendənt ˈveərɪəbl]. The set $\mathcal{D}$ is called the *domain* [dəʊˈmeɪn] of f. If $y = f(x)$, then x is the *pre-image* [priːˈɪmɪdʒ] or *counterimage* [ˌkaʊntəˈrɪmɪdʒ] of y under f and y is called the *dependant* [dɪˈpendənt] *variable*.

To denote a function, we write $f : \mathcal{D} \longrightarrow \mathbb{R};\ x \longmapsto f(x)$ 'f maps $\mathcal{D}$ to $\mathbb{R}$ and x onto $f(x)$'. For example, let f be the function $f : [\,0\,;\,10\,] \longrightarrow \mathbb{R};\ x \longmapsto x^2 + 4$. The image of 2 under f is $f(2) = 2^2 + 4 = 8$. So 8 is the image of 2 under f and 2 is a pre-image of 8 under f. We can also say that 8 is the *value* of f for the *argument* [ˈɑːgjʊmənt] 2.

9.2 Graphs of functions

DEFINITION 9–2:

Let f be a function from $\mathcal{D}$ to $\mathbb{R}$. The *graph* of f in the coordinate system $(O\,;\,\vec{\imath}\,;\,\vec{\jmath}\,)$ is the set, denoted $\mathcal{F}$, of points M the coordinates of which are $(\,x\,;\,f(x)\,)$ with $x \in \mathcal{D}$. The curve $\mathcal{F}$ is said to have equation $y = f(x)$.

PROPOSITION 9–1:

Let $M : (\,x\,;\,y\,)$ be a point in the plane. Let f be a function from $\mathcal{D}$ to $\mathbb{R}$ and $\mathcal{F}$ its graph.

$$M \in \mathcal{F} \Longleftrightarrow x \in \mathcal{D} \textbf{ and } y = f(x).$$

9.3 Increasing, decreasing functions

In this section the domain of the function f is an **interval** I.

DEFINITION 9–3:

Let $f : I \to \mathbb{R}$, $x \longmapsto f(x)$. f is increasing [ɪnˈkriːsɪŋ] on I iff for all pair $(\,a\,;\,b\,)$ of numbers belonging to I $a < b$ implies $f(a) < f(b)$.

So an increasing function is ‘order-preserving’: the images $f(a)$ and $f(b)$ are in the same order as the arguments a and b.

Example:

1. Let $f : \mathbb{R} \to \mathbb{R}$, $x \longmapsto 4x + 5$. Let a and b be two real numbers. $a < b$ implies $4a < 4b$ which in turn implies $4a + 5 < 4b + 5$ i.e. $f(a) < f(b)$. So f is increasing on $\mathbb{R}$.

2. Let $f : \mathbb{R} \to \mathbb{R}$, $x \longmapsto x^2 - 6x$. First we have $f(1) = -5$ and $f(2) = -8$ so $1 < 2$ and $f(1) > f(2)$. That proves that f is not increasing on $\mathbb{R}$.

   We have $x^2 - 6x = (x-3)^2 - 9$. So if $3 \le a < b$ then $0 \le a - 3 < b - 3$ and, as the square of two positive numbers are ordered as the numbers, $(a-3)^2 < (b-3)^2$ which implies $(a-3)^2 - 9 < (b-3)^2 - 9$ i.e. $f(a) < f(b)$. We have proven that f is increasing on $[\,3\,;\,+\infty\,[$.

DEFINITION 9–4:

Let $f : I \to \mathbb{R}$, $x \longmapsto f(x)$. f is decreasing [diːˈkriːsɪŋ] on I iff for all pair $(\,a\,;\,b\,)$ of numbers belonging to I $a < b$ implies $f(a) > f(b)$.

So a decreasing function is 'order-reversing': the images $f(a)$ and $f(b)$ are in the reverse order as the arguments a and b.

Example:

1. Let $f : \mathbb{R} \to \mathbb{R}$, $x \longmapsto -3x + 8$. Let a and b be two real numbers. $a < b$ implies $-3a > -3b$ which in turn implies $-3a+8 > -3b+8$ i.e. $f(a) > f(b)$. So f is decreasing on $\mathbb{R}$.

2. Let $f : \mathbb{R} \to \mathbb{R}$, $x \longmapsto x^2 - 6x$.

   We have proven that f is increasing on $[\,3\,;\,+\infty\,[$.

   Let us cast a glance at the behaviour of f on $]-\infty\,;\,3\,]$. $a \in\ ]-\infty\,;\,3\,]$ iff $a \leq 3$. Let us consider two numbers a and b in $]-\infty\,;\,3\,]$ such that $a < b$. Then $a < b \leq 3$ so $a - 3 < b - 3 \leq 0$ and $(a-3)^2 > (b-3)^2$ which implies $(a-3)^2 - 9 > (b-3)^2 - 9$ i.e. $f(a) > f(b)$. So f is decreasing on $]-\infty\,;\,3\,]$.

DEFINITION 9–5:

Let $f : I \to \mathbb{R}$, $x \longmapsto f(x)$.

1. f is non-increasing on I iff, for all pair $(\,a\,;\,b\,)$ of numbers belonging to I, $a < b$ implies $f(a) \geq f(b)$.

2. f is non-decreasing on I iff, for all pair $(\,a\,;\,b\,)$ of numbers belonging to I, $a < b$ implies $f(a) \leq f(b)$.

9.4 Maxima, minima, turning points

In this section too the domain of the function f is an **interval** I.

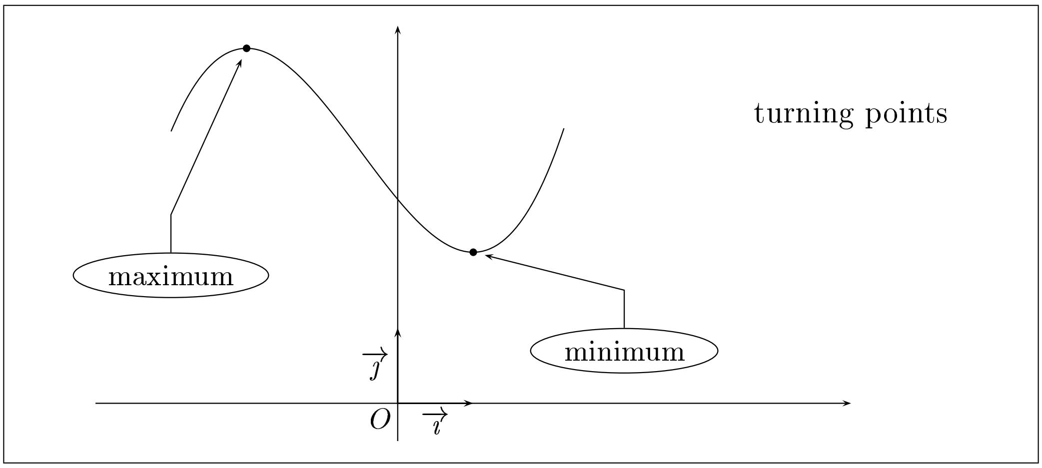
DEFINITION 9–6:

f has a *maximum* [ˈmæksɪməm] at a on I iff, for all $x \in I$, $f(x) \leq f(a)$. Then $f(a)$ is the maximum of f on I.
f has a *minimum* [ˈmɪnɪməm] at a on I iff, for all $x \in I$, $f(x) \geq f(a)$. Then $f(a)$ is the minimum of f on I.

***Remark**: The word 'maximum' [resp. 'minimum'] has two plurals: 'maximums' [resp. 'minimums'] and 'maxima'* [ˈmæksɪmə] *[resp. 'minima'].*

Maxima and minima are also called *turning points*.

Fig. 9–1



9.5 Linear functions

DEFINITION 9–7:

Let a and b be two reals. The function $f : \mathbb{R} \longrightarrow \mathbb{R};\ x \longmapsto f(x)$ is *linear* [ˈlɪnɪəʳ] iff $f(x) = ax + b$.

Remark: *If f is linear with $f(x) = ax + b$ and if $a = 0$ then the function is* constant *[*ˈkɒnstənt] i.e. *all the reals have the same image b.*

PROPOSITION 9–2:

If f is a non-constant linear function, then the increment of y is proportional to the increment of x.

Proof:

Let f be a non-constant linear function such that $f(x) = ax + b$. Let x and x' be two reals. The increment of the independent variable is $x' - x$. The increment of the dependent variable is $f(x') - f(x)$.

We have $f(x') - f(x) = (ax' + b) - (ax + b) = ax' + b - ax - b = ax' - ax = a\,(x' - x)$.

PROPOSITION 9–3: (**converse of the preceding**)

Let f be a function from $\mathbb{R}$ to $\mathbb{R}$. If the increment of y is proportional to the increment of x, then f is a linear function.

PROPOSITION 9–4:

The graph of a linear function is a line.

PROPOSITION 9–5: (**Behaviour of a linear function**)

Let a and b be two reals. Let f be defined by $f : \mathbb{R} \longrightarrow \mathbb{R};\ x \longmapsto f(x) = ax+b$.

- If $a = 0$ then f is constant on $\mathbb{R}$.
- If $a > 0$ then f is increasing on $\mathbb{R}$.

- If $a < 0$ then f is decreasing on $\mathbb{R}$.

***Remark**: In French the linear functions are called **'affine'**. For a function f to be **'linéaire'**, f has to map x onto ax with a a constant real. Such a function has a graph which is a line passing through O the origin of the plane.*
In English too it is possible to use the words 'affine' [əˈfaɪn] and 'linear' with the same meanings as in French but it is usually so just in University texts.

9.6 The 'square' function

Let s be defined by $s : \mathbb{R} \longrightarrow \mathbb{R};\ x \longmapsto x^2$.
PROPOSITION 9–6:
The function s is decreasing on $]-\infty\,;\,0]$ and increasing on $[0\,;\,+\infty[$.
Proof:
Let a and b be two reals.
First let a and b be such that $0 \leq a < b$. Let us determine the sign of $s(b) - s(a)$.

$$s(b) - s(a) = b^2 - a^2 = (b - a)\,(b + a)$$

Now, as $a < b$, $b - a > 0$ and, as $0 \leq a$ and $0 < b$, $b + a > 0$ therefore $s(b) - s(a) > 0$ i.e. $s(a) < s(b)$.
So, s is increasing on $[0\,;\,+\infty[$.
Secondly let a and b be such that $a < b \leq 0$. Then, as $a < b$, $b - a > 0$ and, as $a < 0$ and $b \leq 0$, $b + a < 0$ therefore $s(b) - s(a) < 0$ i.e. $s(a) > s(b)$.
So, s is decreasing on $]-\infty\,;\,0]$. QED

We can sum up the behaviour of the 'square' function with the following table:

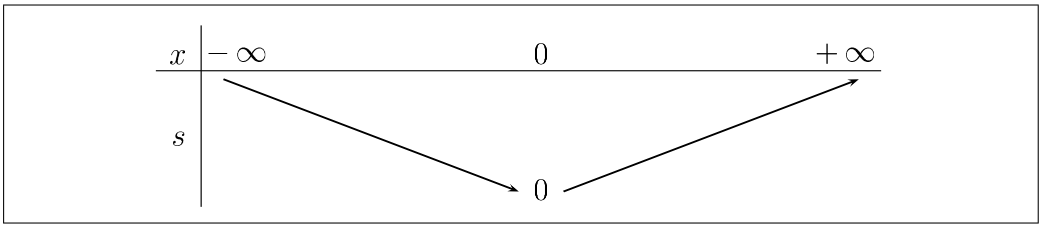


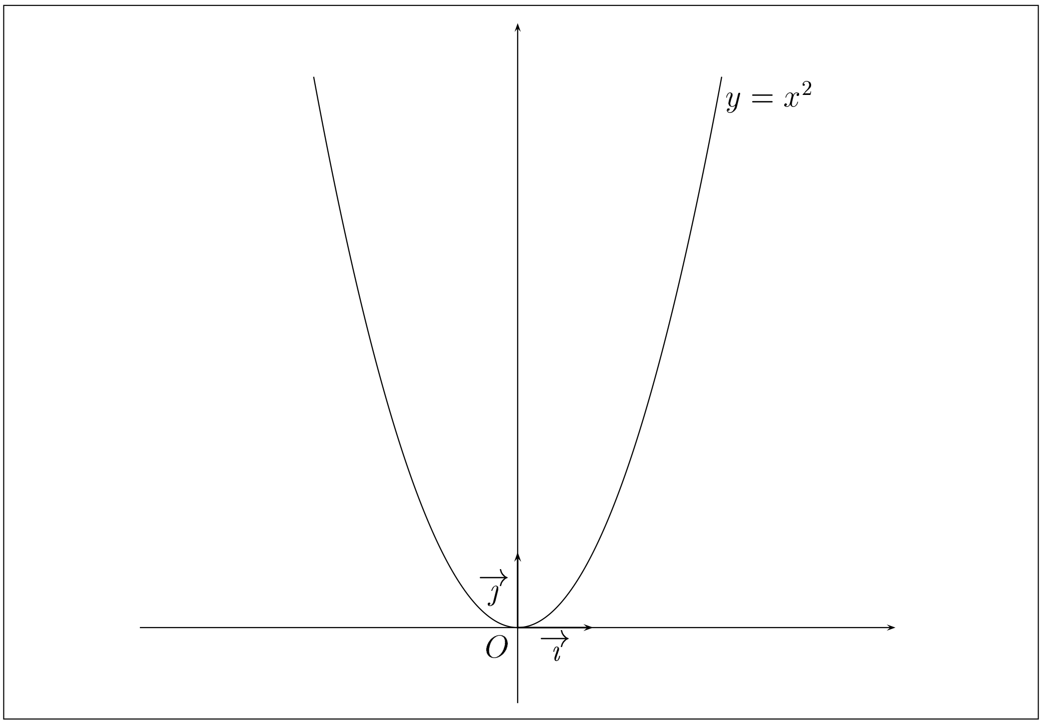
Fig. 9–2

As consequences of the preceding proposition, we can state:
PROPOSITION 9–7:

- The 'square' function has a minimum at 0. That minimum is 0.
- If a and b are negative reals then $a < b$ is equivalent to $a^2 > b^2$.
- If a and b are positive reals then $a < b$ is equivalent to $a^2 < b^2$.

The graph $\mathcal{S}$ of the function s is a *parabola* [pəˈræbələ] with *vertex* at O and the y-axis as line of symmetry. So $\mathcal{S}$ is invariant under the reflection in the y-axis.

Fig. 9–3



PROPOSITION 9–8:
Let a be a positive real ($a > 0$). Then:

- $x^2 = a \iff x = \sqrt{a}$ or $x = -\sqrt{a}$ i.e. the equation $x^2 = a$ has exactly two solutions which are $\sqrt{a}$ and $-\sqrt{a}$;
- $x^2 \leq a \iff x \in \left[-\sqrt{a}\,;\,\sqrt{a}\right]$;
- $x^2 \geq a \iff x \in \left]-\infty\,;\,-\sqrt{a}\right] \cup \left[\sqrt{a}\,;\,+\infty\right[$.

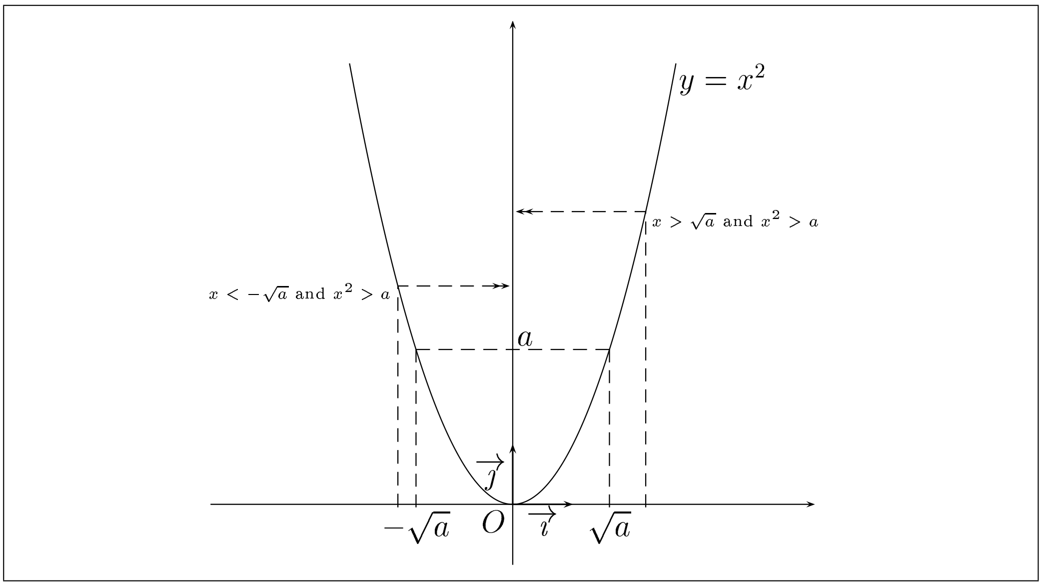


Fig. 9–4

9.7 The 'inverse' function

Let i be defined by $i : \mathbb{R}^* \longrightarrow \mathbb{R};\ x \longmapsto \dfrac{1}{x}$.

PROPOSITION 9–9:

The function i is decreasing on $]-\infty\,;\,0\,[$ and on $]\,0\,;\,+\infty\,[$.

Proof:

Let a and b be two reals not equal to 0. We have

$$i(b) - i(a) = \frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$$

now, if $a < b$ then $a - b < 0$, therefore the sign of $i(b) - i(a)$ is minus times the sign of ab. If, on one hand, $a < b < 0$ then $ab > 0$ and $i(b) - i(a) > 0$, and if, on the other hand, $0 < a < b$ then $ab > 0$ again and $i(b) - i(a) > 0$. Therefore whether a and b are both negative or positive, if $a < b$ then $i(a) > i(b)$ i.e. i is decreasing on both $]-\infty\,;\,0\,[$ and $]\,0\,;\,+\infty\,[$. QED

We can sum up the behaviour of the 'inverse' function with the following table:

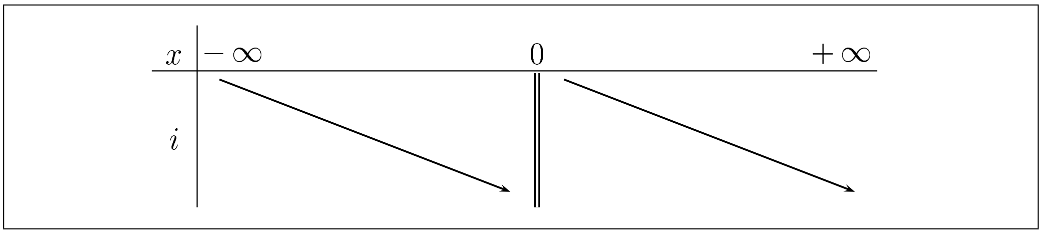


Fig. 9–5

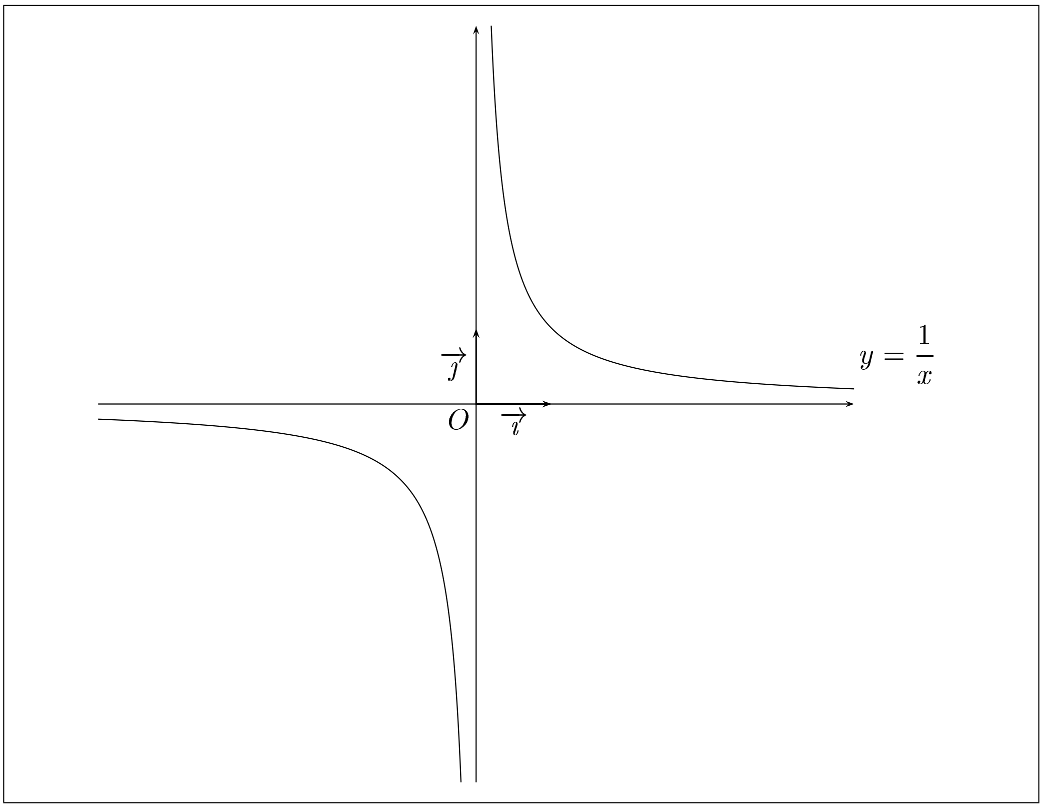
PROPOSITION 9–10:

- If a and b are negative $(a < 0$ and $b < 0)$ then $a \leq b \Longleftrightarrow \dfrac{1}{a} \geq \dfrac{1}{b}$.

- If a and b are positive $(a < 0$ and $b < 0)$ then $a \leq b \Longleftrightarrow \dfrac{1}{a} \geq \dfrac{1}{b}$.

The graph $\mathcal{I}$ of the function i is an *hyperbola* [haɪˈpɜːbələ] with the x-axis and the y-axis as 'axes' or 'asymptotes'. $\mathcal{I}$ is invariant under the reflection in 0.

Fig. 9–6

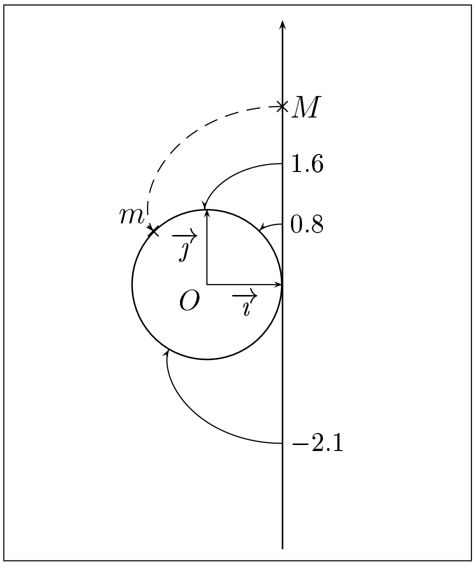


9.8 The trigonometric functions

Among the *trigonometric* [ˌtrɪgənəˈmetrɪk] *functions* or *circular* [ˈsɜːkjʊləʳ] *functions* are the *sine* [saɪn] and the *cosine* [ˈkəʊsaɪn].

9.8.1 Definitions

Let $(O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath}\,)$ be a coordinate system in the plane. Let I and J be points in the plane such that $\overrightarrow{OI} = \overrightarrow{\imath}$ and $\overrightarrow{OJ} = \overrightarrow{\jmath}$. Let the triangle IOJ be positive. Let $\mathcal{C}$ be a directed circle with center O and radius 1 and $\mathcal{D}$ be the tangent to $\mathcal{C}$ at I. Let A be a point on $\mathcal{D}$ such that $IA = 1$ and A and J are on the same side of (OI). Let x be a real. There exists one and only one point M on $\mathcal{D}$ such that x is the abscissa of M relatively to $(\,I\,;\,A\,)$. If we wind $\mathcal{D}$ round $\mathcal{C}$ as shown on the figure, M coincides with a point m on $\mathcal{C}$. Then



Definition 9–8:

- The cosine of x, denoted by $\cos x$, is the abscissa of m in $(\,O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath}\,)$.
- The sine of x, denoted by $\sin x$, is the ordinate of m in $(\,O\,;\,\overrightarrow{\imath}\,;\,\overrightarrow{\jmath}\,)$.

The function *sine* is the function which associates $\sin x$ to any real x. The function *cosine* is the function which associates $\cos x$ to any real x.

9.8.2 Graphs

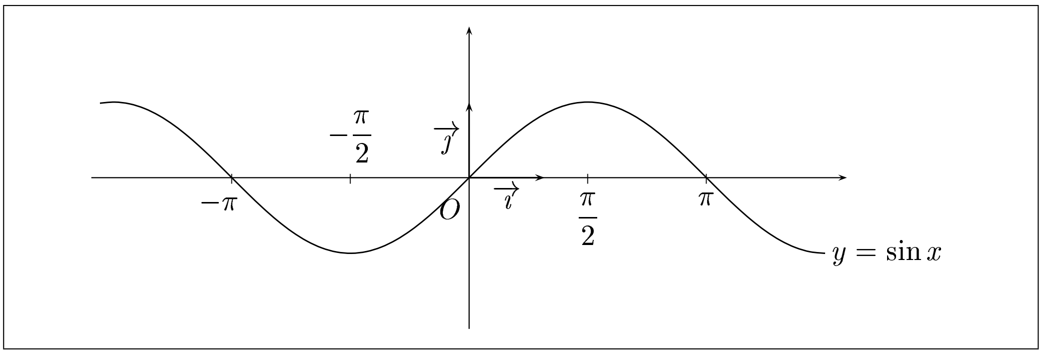
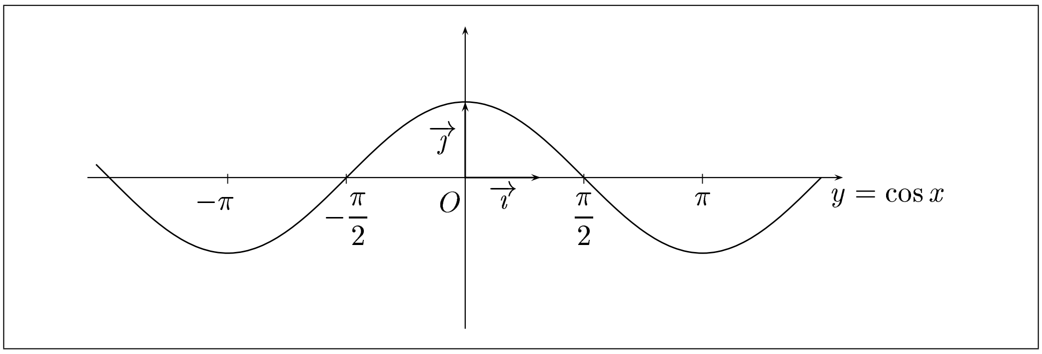


Fig. 9–7

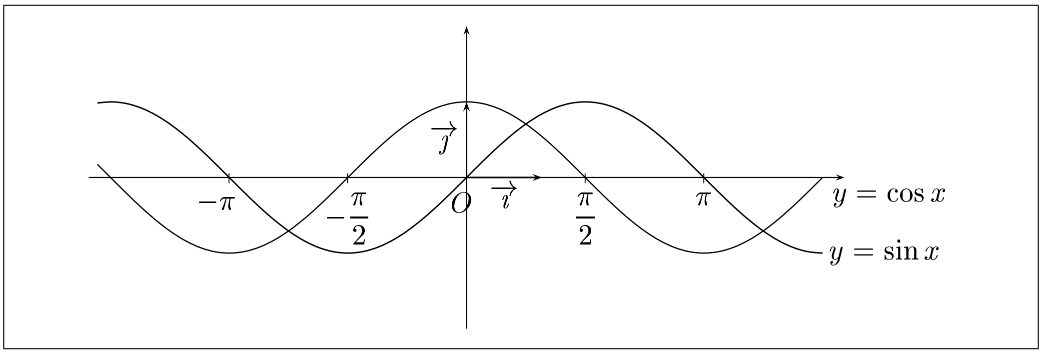
***Remark**: We observe, and we could prove, that the graph of sine is symmetrical about O the origin of the coordinate system. That means that, for all real x,* $\sin(-x) = -\sin x$. *The function sine is said to be* odd.

Fig. 9–8



Remark: *We observe, and we could prove, that the graph of cosine is symmetrical about the y-axis. That means that, for all real x, $\cos(-x) = \cos x$. The function cosine is said to be* even.

Fig. 9–9



Remark: *We observe, and we could prove, that the graph of cosine is the image of the graph of sine under the translation by vector $\frac{\pi}{2}\overrightarrow{\imath}$.*
For all x real, $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$. The functions sine and cosine are thus said to be periodic [ˌpɪərɪˈɒdɪk] *with* period [ˈpɪərɪəd] 2π.

9.8.3 The radian

The *radian* [ˈreɪdɪən] is a unit of measurement of angles. Its abbreviation is 'rad'. An angle between two radii [1] [ˈreɪdɪˌaɪ] that cut off on the circumference of a circle an arc equal in length to the radius has a measure of 1 rad. A right angle measures $\frac{\pi}{2}$ rad.

[1] *plural of* radius [ˈreɪdɪəs].

9.8.4 Some remarkable values

x (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

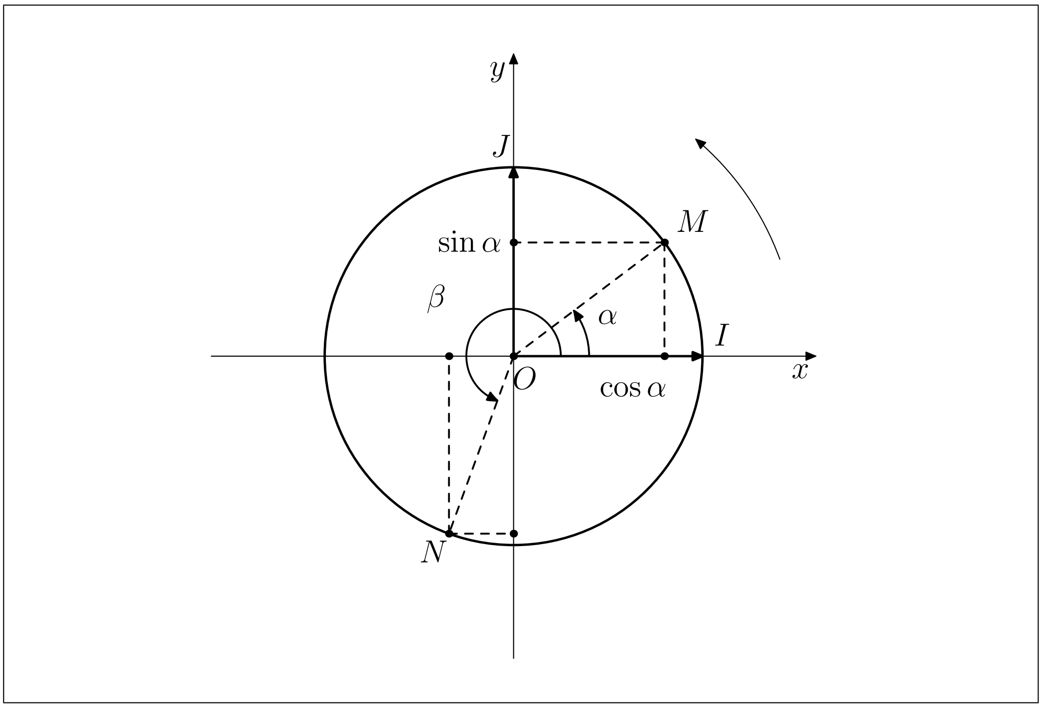


Fig. 9–10

Appendix A

Miscellanea

A.1 Transcription phonétique

J'ai suivi la transcription proposée par [13]. Je la redonne ici telle qu'elle est détaillée en page xiii :

Voyelles

bean	barn	born	boon	burn				
iː	ɑː	ɔː	uː	ɜː				
pit	pet	pat	putt	pot	put	another		
ɪ	e	æ	ʌ	ɒ	ʊ	ə – ə		
bay	buy	boy	no	now	peer	pair	poor	(pour)
eɪ	aɪ	ɔɪ	əʊ	aʊ	ɪə	eə	ʊə	ɔə

On lit en note que la paire /ɛə/ a été simplifiée en /eə/.

Consonnes

pin	bin	tin	din	come	gum	chain	Jane
p	b	t	d	k	g	tʃ	dʒ
fine	vine	think	this	zeal	sheep	measure	how
f	v	θ	ð	z	ʃ	– ʒ	h
sum	sun	sung	light	right	wet	yet	
– m	– n	– ŋ	l	r	w	j	

On note l'accent tonique principal avec « ˈ » et l'accent secondaire avec « ˌ ». Un petit « r » en exposant « ʳ » note l'apparition éventuelle d'un « r » à la liaison.

Lorsque un ə est placé entre parenthèses, il peut être amuï, c'est-à-dire, par exemple, que [p(ə)l] peut se lire [pəl] ou bien [pl].

On donne à côté des mots *au singulier ou au pluriel* la prononciation du singulier.

On trouvera dans l'annexe D « hypothesis *plur.* -ses [haɪˈpɒθɪsɪs -siːz] » pour indiquer que le pluriel de « hypothesis [haɪˈpɒθɪsɪs] » est « hypotheses [haɪˈpɒθɪsiːz] ». On n'y a indiqué que les pluriels **irréguliers**.

Lorsque [13] donne plusieurs prononciations possibles pour un même mot dans un même champ lexical, je n'ai, en général, donné que la prononciation principale. À défaut de trouver la prononciation d'un mot dans [13], j'ai utilisé [21] ou encore [26].

A.2 Greek alphabet

lower-case	capital	english	[pron.]	french
α	A	alpha	[ˈælfə]	alpha
β	B	beta	[ˈbiːtə]	bêta
γ	Γ	gamma	[ˈgæmə]	gamma
δ	Δ	delta	[ˈdeltə]	delta
ε	E	epsilon	[epˈsaɪlən]	epsilon
ζ	Z	zeta	[ˈziːtə]	dzéta
η	H	eta	[ˈiːtə]	êta
θ	Θ	theta	[ˈθiːtə]	thêta
ι	I	iota	[aɪˈəʊtə]	iota
κ	K	kappa	[ˈkæpə]	kappa
λ	Λ	lambda	[ˈlæmdə]	lambda
μ	M	mu	[mjuː]	mu
ν	N	nu	[njuː]	nu
ξ	Ξ	xi	[ksaɪ]	xi [ksi]
o	O	omicron	[əʊˈmaɪkrən]	omicron
π	Π	pi	[paɪ]	pi
ρ	P	rho	[rəʊ]	rô
σ	Σ	sigma	[ˈsɪgmə]	sigma
θ	T	tau	[tɔː]	tau
υ	Υ	upsilon	[juːˈpsaɪlən]	upsilon
φ	Φ	phi	[faɪ]	phi [fi]
χ	X	chi	[kaɪ]	khi
ψ	Ψ	psi	[saɪ]	psi
ω	Ω	omega	[ˈəʊmɪgə]	oméga

ϵ, ϑ, ϖ, ϱ, and ϕ are variants of ε, θ, π, ρ and φ respectively.
σ is written ς at the end of a word.

Appendix B

Geometry

B.1 Plane geometry

> There is no 'royal road' to geometry.
> EUCLID [1] (4th–3rd c. BC) Greek mathematician
> Adressed to Ptolemy I

The plane [pleɪn] geometry [dʒɪˈɒmətrɪ] deals with figures such as points [pɔɪnt], lines [laɪn], triangles [ˈtraɪæŋgl], circles [ˈsɜːkl], quadrilaterals [ˌkwɒdrɪˈlæt(ə)r(ə)l]. . . which lie in a plane. You can think of a plane as a flat surface which extends infinitely [ˈɪnfɪnətlɪ] in all direction. One can represent a plane by a piece of paper or a blackboard.

B.1.1 Points and lines

Given two distinct points A and B in the plane, there is one and only one line which goes or passes through those points. We denote that line by (AB). The point A is *on the line* (AB). We can denote that fact by $A \in (AB)$. If points are on a same line, they are said to be *collinear* [kɒˈlɪnjəʳ].

The line segment or segment between A and B is the set consisting of A, B and all the points on the line (AB) lying between A and B. We denote this segment by $[AB]$. The length of the segment $[AB]$ is also the distance from A to B. It is denoted by AB.

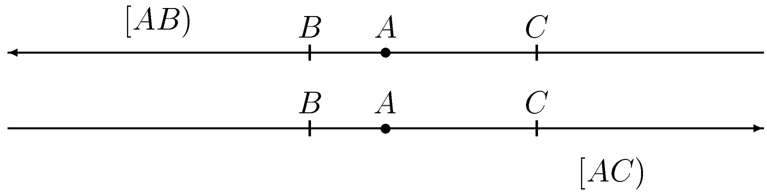
[1] EUCLID [ˈjuːklɪd] taught in Alexandria [ˌælɪˈgzɑːndrɪə] circa [ˈsɜːkə] 300 BC, and was probably the founder of its mathematical school. His chief extant work is the 13-volume *Elements*, which became the most widely known mathematical book of classical antiquity.

The *midpoint* [ˈmɪdpɔɪnt] I of the segment $[AB]$ is the point such that I is on $[AB]$ and $AI = IB$.

Let $\mathcal{L}$ and $\mathcal{L}'$ be two lines. If $\mathcal{L} = \mathcal{L}'$ or if $\mathcal{L}$ and $\mathcal{L}'$ have no common point, $\mathcal{L}$ and $\mathcal{L}'$ are *parallel* [ˈpærəlel]. If $\mathcal{L}$ and $\mathcal{L}'$ are not parallel, they meet in exactly one point. They cut each other. If A is the common point of $\mathcal{L}$ and $\mathcal{L}'$, we can say that $\mathcal{L}$ and $\mathcal{L}'$ intersect [ˌɪntəˈsekt] at A. A is their *intersection point* [ˌɪntəˈsekʃn].

Let A be a point on the line $\mathcal{L}$. A defines on $\mathcal{L}$ two *rays* [reɪ] or *half lines* starting from A. If B and C are on $\mathcal{L}$ such that A is on $[BC]$, we can denote those rays by $[AB)$ and $[AC)$ respectively. A is called the *vertex* [ˈvɜːteks] of the rays $[AB)$ and $[AC)$.

Fig. B–1



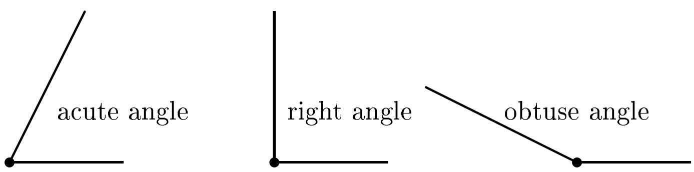
B.1.2 Angles

Let A, B and C be three collinear points such that $B \in [AC]$.They define a *straight angle* [streit æŋgl] viz.[2] $\angle ABC$ ‘angle ABC’. The angle $\angle ABA$ is said to be *full*.

Two rays with the same vertex separate the plane into two regions. Each one of these regions together with the rays is called an *angle* determined by the rays, e.g. $\angle ABC$ is determined by the rays $[BA)$ and $[BC)$. Those rays are called the *legs* [leg] of the angle ABC.

An angle can be *acute* [əˈkjuːt], *right* [raɪt] or *obtuse* [əbˈtjuːs].

Fig. B–2



Two angles are *adjacent* [əˈdʒeɪsənt] if they have a ray in common.

We can measure angles with *degrees* [dɪˈgriː]. A right angle is an angle of 90°. A straight angle has 180°. If the sum of two angles is equal to

[2] viz. [vɪz] or [ˈneɪmlɪ]: namely; that is to say; in other words. (abbr. of the Latin VIDELICET, *z* being a medieval Latin symbol for abbreviation of *-et*)

90 ˚ , those angles are said to be *complementary* [ˌkɒmplɪˈmentərɪ]. If their sum is equal to 180 ˚ , they are said to be *supplementary* [ˌsʌplɪˈmentərɪ]. To measure an angle we use a *protractor* [prəˈtræktəʳ].

We define the *angle bisector* to be the ray which divides the angle into two adjacent angles having the same measure.

We define two lines to be *perpendicular* [ˌpɜːpenˈdɪkjʊləʳ] if they intersect, and if the angle between the lines is a right angle. The *mid-perpendicular* or *perpendicular bisector* of segment $[AB]$ is the line which passes through the midpoint of $[AB]$ and which is perpendicular to (AB).

On the figure below, $\angle CAB$ and $\angle BAD$ are adjacent and supplementary; $\angle CAB$ and $\angle EAD$ are said to be *vertical* [ˈvɜːtɪkl]; $\angle CAB$ and $\angle FEA$ are said to be *corresponding* [ˌkɒrɪsˈpɒndɪŋ]; $\angle CAB$ and $\angle GEH$ are *alternate* [ɔːlˈtɜːnət].

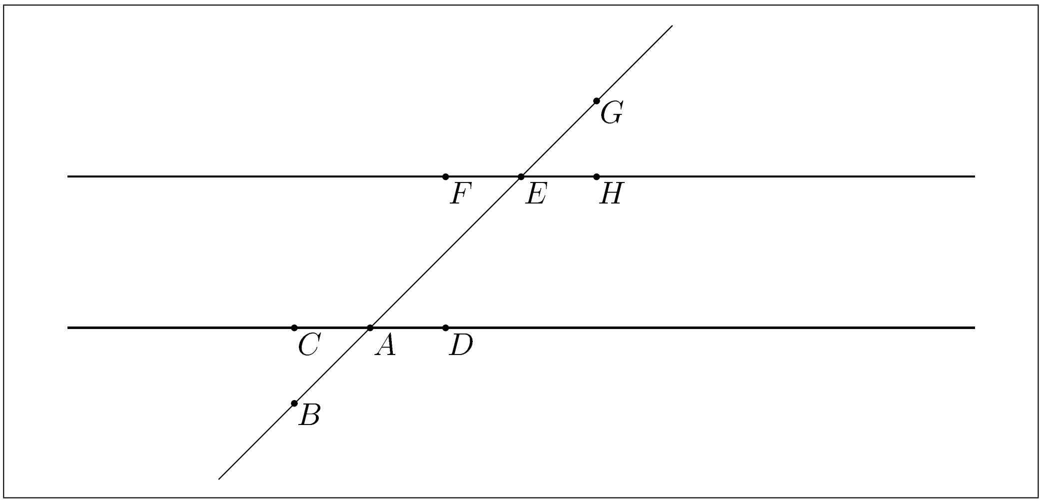


Fig. B–3

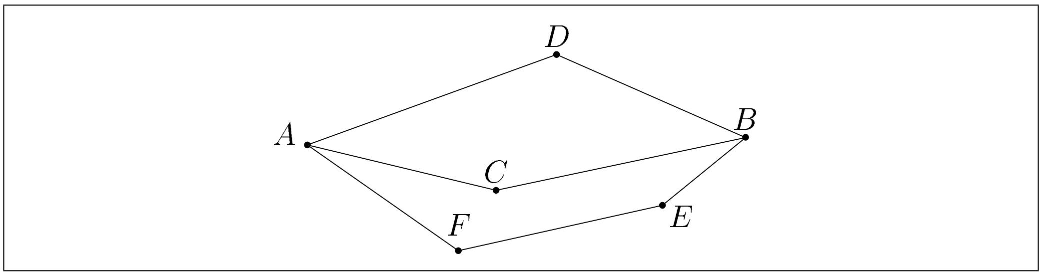
Line (AE) is called a *transversal* [trænzˈvɜːsl] or *traverse* [ˈtrævəs] for it intersects two (or more) lines.

B.1.3 Polygons

A *polygon* [ˈpɒlɪgən] is a closed plane figure bounded by three or more straight line segments that terminate in pairs at the same number of vertices, and do not intersect other than at their vertices. A polygon is *regular* [ˈregjʊləʳ] if it has all its sides and all its angles equal. Specific polygons have names that indicate the number of sides such as triangle, quadrilateral, *pentagon* [ˈpentəgən] — five sides —, *hexagon* [ˈheksəgən] — six sides —, and so on.

A polygon is *convex* [ˈkɒnveks] if it has no interior angle greater than 180 ˚ . On the figure below $ADBEF$ is convex whereas $ACBEF$ is not. A polygon which is not convex is said to be *concave* [ˈkɒŋkeɪv].

Fig. B–4



B.1.4 Triangles

A triangle is a three-sided plane figure. Three non-collinear points determine a triangle. If A, B and C are not collinear, they are the *vertices* [ˈvɜːtɪsiːz] (plural of 'vertex' [ˈvɜːteks]) of the triangle ABC. Triangle ABC can be denoted by $\triangle ABC$.

Triangles can be classified by the relative [ˈrelətɪv] lengths of their sides. A *scalene* [ˈskeɪliːn] triangle has no sides of equal length; an *isosceles* [aɪˈsɒsɪliːz] triangle has at least two equal sides; an *equilateral* [ˌiːkwɪˈlæt(ə)r(ə)l] triangle has three equal sides. A *right* or *right-angled* triangle has one angle of 90°.

Pythagoras'[3] theorem [paɪˈθægəræs ˈθɪərəm] (also known as Pythagorean [paɪˌθægəˈrɪən] theorem) states that if $\triangle ABC$ is a right triangle with *legs* of lengths a and b, and *hypotenuse* [haɪˈpɒtənjuːz] of length c, then $c^2 = a^2 + b^2$.

An *altitude* [ˈæltɪtjuːd] is a segment between a vertex and the opposite [ˈɒpəzɪt] side or *base* [beɪs], that is perpendicular to the side. It is also the length of that segment. The common point of the three altitudes is the *orthocentre* [ˌɔːθəʊˈsentəʳ].

A *median* [ˈmiːdjən] is a line joining a vertex of a triangle to the midpoint of the opposite side. All three such lines coincide in the *centroid* [ˈsentrɔɪd]. Archimedes [ˌɑːkɪˈmiːdiːz] (c. 287–212 B.C.) obtained it as the center of gravity [ˈgrævɪtɪ] of a triangular [traɪˈæŋgjʊləʳ] plate of uniform density [ˈdensətɪ]. The centroid of the triangle is also the point of trisection [traɪˈsekʃn] of the three medians of the triangle.

The mid-perpendiculars of the sides of a triangle intersect at the *cir-*

[3]Pythagoras (6th-c BC) Philosopher and mathematician, born in Samos [ˈseɪmɒs], Greece. He settled at Crotona, Magna Graecia [ˌmægnə ˈgriːsɪə] (c. 530 BC) where he founded a moral and religious school. He eventually fled from ther because of persecution, settling at Megapontum in Lucania [luːˈkeɪnɪə]. The famous theorem attributed to him was probably developed later by members of the Pythagorean school, which is best known for its studies of the relations between numbers.

cumcentre [ˌsɜːkəmˈsentəʳ] which is the *centre* [ˈsentəʳ] of the *circumcircle* [ˈsɜːkəmˌsɜːkl] or *circumscribed* [ˈsɜːkəmˌskraɪbd] *circle*.

B.1.5 Trigonometry

Trigonometry [ˌtrɪgəˈnɒmətrɪ] is the branch of mathematics concerned with the properties of the *trigonometric* [ˌtrɪgənəˈmetrɪk] *ratios* [ˈreɪʃɪəʊ] or *trig ratios* and their application to the determination of the sides and angles of triangles, used in surveying [səˈveɪɪŋ], navigation…

To survey : [səˈveɪ] **1** take or present a general view of. **2** examine the condition of (a building etc.), especially on behalf of a prospective buyer. **3** determine the boundaries, extent, ownership, etc., of (a district etc.).

The trig ratios are: *sine* [saɪn], *cosine* [ˈkəʊsaɪn], *tangent* [ˈtændʒənt], cotangent [ˌkəʊˈtændʒənt], secant [ˈsiːkənt], and cosecant [ˌkəʊˈsiːkənt].

Let ABC be a right triangle the hypotenuse of which is AB. Let α denote $\angle BAC$. Then

$$\sin\alpha = \frac{BC}{AB} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\alpha = \frac{AC}{AB} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\alpha = \frac{BC}{AC} = \frac{\text{opposite}}{\text{adjacent}}$$

There are several mnemonics [niːˈmɒnɪk] (memory aids) which can help you to remember those formulae[4] [ˈfɔːmjʊliː]. Here is an example:

S O H C A T H T O A
Some Officers Have Curly Auburn Hair To Offer Attraction.

S stands for ‘sine’, C for ‘cosine’, T for ‘tangent’, A for ‘adjacent’, H for ‘hypotenuse’, and O for ‘opposite’.

B.1.6 Quadrilaterals

A *quadrilateral* or *trapezoid* [ˈtræpɪzɔɪd] is any four-sided plane figure. It has four vertices. The *diagonals* [daɪˈægənl] of $ABCD$ are $[AC]$ and $[BD]$.

A *trapezium* [trəˈpiːzjəm] is a quadrilateral with two parallel sides of unequal length. A *parallelogram* [ˌpæræˈleləgræm] is a quadrilateral with opposite pairs of sides parallel. A *rhombus*[5] [ˈrɒmbəs] is an equilateral parallel-

[4] plural of ‘formula’ [ˈfɔːmjʊlə]

[5] plural: ‘rhombi’ [ˈrɒmbaɪ] or ‘rhombuses’ [ˈrɒmbəsɪz].

ogram — all its sides are equal. A *rectangle* [ˈrekˌtæŋgl] is a quadrilateral all the angles of which are right. A *square* [skweəʳ] is an equilateral rectangle.[6]

If the lengths are measured with metres [ˈmiːtəʳ], the *area* [ˈeərɪə] are measured with square metres. The area of a square of side s m is equal to s^2 m^2 'square meter'.

B.1.7 Circles

A *circle* is a closed [kləʊzd] plane *curve* [kɜːv] every point of which is equidistant from a given fixed point, the *centre*. In other words, a circle is the *path* [pɑːθ] of a point that moves so as to keep a constant distance from a fixed point, the centre.

Each circle comprises [kəmˈpraɪzɪz] a *radius*[7] [ˈreɪdɪəs] — the distance from any point on the circle to the centre —, a *circumference* [səˈkʌmf(ə)r(ə)ns] — the boundary of the circle —, *diameters* [daɪˈæmɪtəʳ] — lines crossing the circle through the centre —, *chords* [kɔːd] — lines joining two points on the circumference —, and *tangents* [ˈtændʒənt] — lines that touch the circumference at one point.

An *arc* [ɑːk] is a section of a curved line. A circle has three types of arc: a *semicircle* [ˈsemɪˌsɜːkl], which is exactly half of the circle; *minor arcs* [ˈmaɪnəʳ], which are less than the semicircle; and *major arcs* [ˈmeɪdʒəʳ], which are greater than the semicircle.

The ratio of the distance all around the circle — the *circumference* or *perimeter* [peˈrɪmɪtəʳ] — to the diameter is an irrational number called π [paɪ].

B.2 Solid geometry

Solid [ˈsɒlɪd] geometry is the branch of geometry concerned with the properties of three-dimensional figures. Among these figures there are: the planes, the *cubes*, the *cuboids* [ˈkjuːbɔɪd] or *right parallelepipeds* [ˈpærəˌleləˈpaɪpɪd], the *cylinders* [ˈsɪlɪndəʳ], the *pyramids* [ˈpɪrəmɪd], the *spheres* [sfɪəʳ].

A *polyhedron* [ˌpɒlɪˈhiːdrən][8] is a solid figure. A polyhedron is said to be *convex* if all segments joining any two points on its boundary lie wholly inside it. A regular convex polyhedron or '*platonic solid*' [pləˈtɒnɪk] is a

[6] In US English the British 'trapezoid' is called 'trapezium' and vice versa [ˌvaɪsɪˈvɜːsə].
[7] plural 'radiuses' [ˈreɪdɪəsɪz] or 'radii' [ˈreɪdɪˌaɪ]
[8] plurals: 'polyhedrons' [ˌpɒlɪˈhiːdrənz] or 'polyhedra' [ˌpɒlɪˈhiːdrə]

convex polyhedron the faces of which are regular polygons. There are only five platonic solids *viz* the regular *tetrahedron* [ˌtetrəˈhiːdrən], the cube, the regular *octahedron* [ˌɒktəˈhiːdrən], *icosahedron* [ˌaɪkəsəˈhiːdrən] and *dodecahedron* [ˌdəʊdəkəˈhiːdrən].

The cube is a polyhedron which has eight corners — or 'vertices' — twelve *edges* [edʒ] and six faces. Each face is a square. All the edges of a cube have the same length.

A cuboid is a polyhedron the faces of which are rectangles. It has eight vertices, twelve edges and six faces. If one wishes, one can use the more tongue-twisting words 'right parallelepiped' to speak about a cuboid.

A *net* is a diagram of a hollow solid consisting of the plane shapes of the faces so arranged that the diagram could be folded to form the solid.

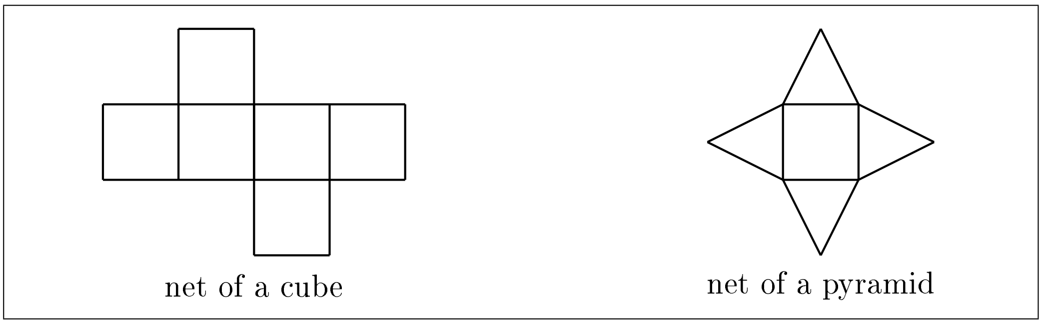


Fig. B–5

B.3 Coordinate system

Once a unit length is selected, we can represent points on a line by numbers.

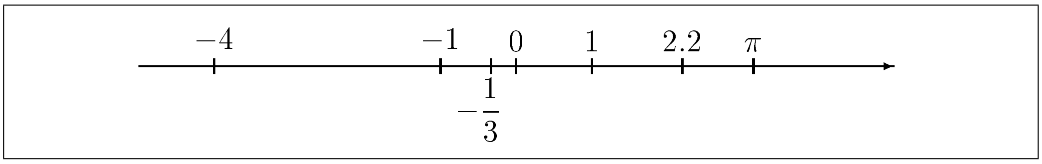


Fig. B–6

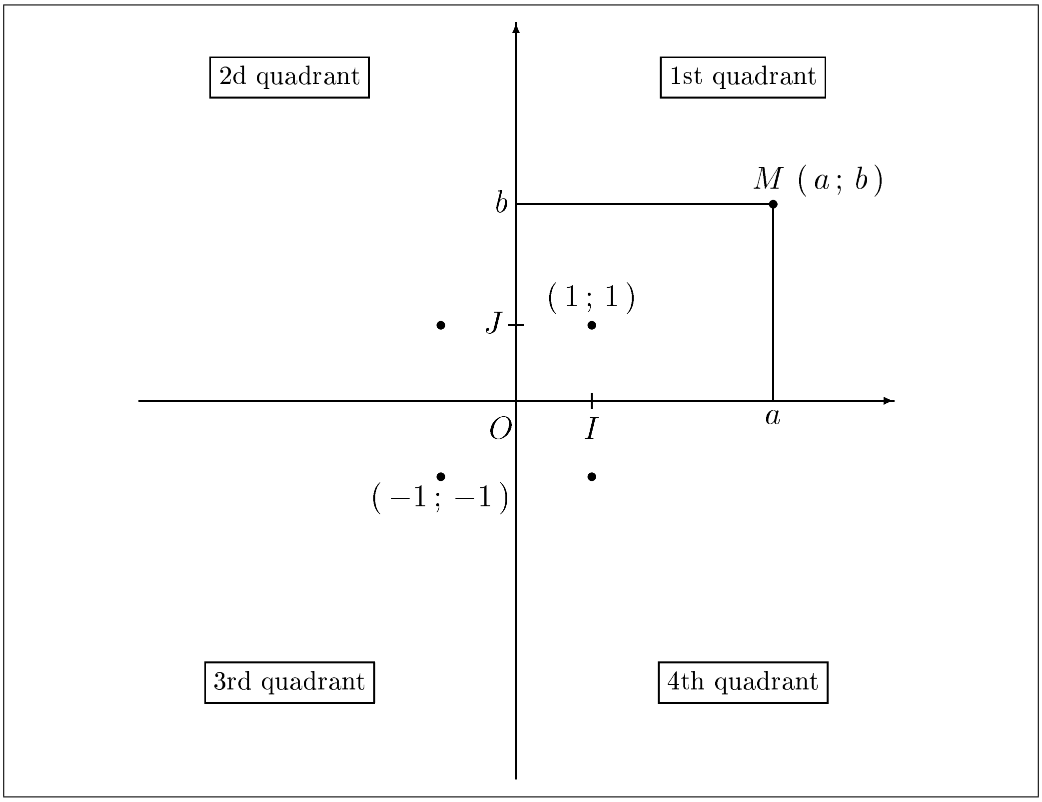
A *coordinate system* [kəʊˈɔːdnət] is any system for locating points by their coordinates with respect to some set of reference [ˈrefrəns] points, lines, directions, etc.

Cartesian coordinates [kæˈtiːzjən] or *rectangular coordinates* [rekˈtæŋgjʊləʳ] provide a system for the representation of a point in plane in terms of its distance, measured along a pair of mutually perpendicular *axes* [ˈækiːz], from a given *origin* [ˈɒrɪdʒɪn]. On the *Cartesian plane* the point $(a\,;\,b)$ is located by measuring a units along the x-axis [ˈæksɪs] and b units along the y-axis, and then finding the point of intersection of the perpendiculars to the axes at those points (see figure below); a is then the *abscissa* [æˈbsɪsə] [9] and

[9] plurals: abscissas [æˈbsɪsəz] or abscissae [æˈbsɪsiː]

b the *ordinate* [ˈɔːdnət]. By convention, the positive directions of the axes point to the right and upwards, so that the four points (±1 ; ±1) are placed as shown; by convention, the first *quadrant* [ˈkwɒdrənt] is that in which both coordinates are positive, and the other quadrants are numbered anticlockwise [ˌæntɪˈklɒkwaɪz] [10] from the first.

Fig. B–7



Cartesian : of or relating to DESCARTES or his work in philosophy, science, and mathematics.

Descartes : René (1596–1650) French philosopher, mathematician, and man of science, often called the father of modern philosophy. In mathematics he developed the use of coordinates to locate a point in two or three dimensions: this enabled the techniques of algebra and calculus to be used to solve geometrical problems.

[10] In a curve opposite in direction to the movement of the hands of a clock.

Appendix C

Units

He gave man speech, and speech created thought
Which is the measure of the universe.
Percy Bysshe SHELLEY (1792–1822), English poet,
in *Prometheus Unbound*[1] (1820)

C.1 The British System

Imperial : [ɪmˈpɪərɪəl] belonging to the official British series of weights [weɪt] and measures [ˈmeʒə] — (of non-metric weights and measures) used or formely used by statute [ˈstætjuːt] i.e. *a written law passed by a legislative body,* e.g. *an Act of Parliament*, in the UK (e.g. *imperial gallon*).

C.1.1 Linear measure

Linear : [ˈlɪnɪə^r] related to length.

1 inch	[ɪn(t)ʃ]	1 in.
1 foot = 12 in.	[fʊt / fiːt]	1 ft.
1 yard = 3 ft.	[jɑːd]	1 yd.
1 (statute) mile = 1,760 yd.	[maɪl]	1 mi.
1 int. nautical mile	[ˈnɔːtɪkl]	

$$1 \text{ in} = 25.4 \text{ mm.} \qquad 1 \text{ international nautical mile} = 1,851.9962 \text{ m.}$$

Nautical distances are measured with fathom [ˈfæðəm] (6 ft) and cable [ˈkeɪbl] (120 fathoms) too.

[1] Percy [ˈpɜːsɪ] Bysshe [bɪʃ] Shelley [ˈʃelɪ] — Prometheus [prəˈmiːθjuːs]

C.1.2 Square measure

One square [skweəʳ] inch is the area of a square the sides of which are 1 inch long. Acres [ˈeɪkəʳ] are used to measure land.

1 square inch	1 sq. in.
1 square foot = 144 sq. in.	1 sq. ft.
1 square yard = 9 sq. ft.	1 sq. yd.
1 acre = 4,840 sq. yd.	
1 square mile = 640 acres	1 sq. mi.

C.1.3 Cubic measure

One cubic [ˈkjuːbɪk] inch is the volume of a cube [kjuːb] the sides of which are 1 inch long.

1 cubic inch	1 cu. in.
1 cubic foot = 1,728 cu. in.	1 cu. ft.
1 cubic yard = 27 cu. ft.	1 cu. yd.

C.1.4 Capacity measure

Measure of capacity : [kəˈpæsɪtɪ] a measure used for vessels [vesl] and liquids [ˈlɪkwɪd] or grains etc.

Vessel : **1** a hollow [hɒˈləʊ] receptacle [rɪˈseptəkl] especially for liquid e.g. a cask, cup, pot, bottle, or dish. **2** a ship or boat, especially a large one.

1 fluid ounce	[ˈfluːɪd aʊns -iz]	1 fl. oz.
1 gill = 5 fluid oz.	[dʒɪl]	1 gi.
1 pint = 20 fluid oz.	[paɪnt]	1 pt.
1 quart = 2 pt.	[kwɔːt]	1 qt.
1 gallon = 4 qt. = 277.42 cu. in.	[ˈgælən]	1 gal.
1 peck = 2 gal.	[pek]	1 pk.
1 bushel = 4 pk.	[ˈbʊʃl]	1 bu.
1 quarter = 8 bu.	[kwɔːtəʳ]	

☺ There exists also 'gill' pronounced [gɪl]. We find in [21]: **1** the respiratory organ in fishes [. . .] **2** the vertical radial plates on the underside of mushrooms [. . .] **3** the flesh below a person's jaws and ears (*green about the gills*). **4** the wattles of fowls.

C.1.5 Avoirdupois weight

Avoirdupois : [ˌævədəˈpɔɪz]: **1**(in full **avoirdupois weight**) a system of weights based on a pound of 16 ounces or 7,000 grains. **2** weight, heaviness.

1 grain	[greɪn]	1 gr *or* 1 gr.
1 dram *or* drachm	[dræm]	1 dr.
1 ounce = 16 dr.	[aʊns]	1 oz
1 pound = 16 oz = 7,000 gr.	[paʊnd -z]	1 lb
1 stone = 14 lb	[stəʊn]	1 st.
1 quarter = 2 st.		1 qt.
1 hundredweight = 4 qt.	[ˈhʌndrədweɪt]	1 cwt
1 (long) ton = 20 cwt	[tʌn]	1 t.
1 short ton = 2,000 lb		

1 lb = $0.453, 592, 37$ kg

C.2 US Customary Weights and Measures

[Excerpt from Encyclopedia Americana (1993) [1]]

It is sometimes thought that the customary system of weights and measures in the British Commonwealth countries and that in the United States are identical. It is true that the US and the British inch are defined identically for scientific work (1 inch = 2.54 centimeters exactly); that the two systems are practically identical in commercial usage; and that many relationships, such as 12 inches = 1 foot, are the same in both systems. However, there are some very important differences.

In the British system an avoirdupois ounce of water at 62° F has a volume of 1 fluid ounce. This convenient relation does not exist in the US system.

Among other differences between the British and the American systems of weights and measures it should be noted that the use of the troy [trɔɪ] pound was abolished in England on Jan. 6th 1879, only the troy ounce and its subdivisions being retained. The troy pound is still legal in the USA, although it is not now greatly used. The common use in England of the stone of 14 pounds should be mentioned. This unit is not now used in the USA. In the apothecaries' [əˈpɒθək(ə)rɪz] systems of liquid measure the British insert a unit, the fluid scruple [ˈskruːpl], equal to one third of a fluid drachm (dram) between their minim [ˈmɪnɪm] and their fluid drachm.

Customary units of measurement continue to be used in almost all aspects of everyday life in the United States, and few indications of popular interest in change are visible. By 1980 none of the 50 states of the United States had enacted legislation requiring the mandatory use of International Units. However, many of them had modified their laws to accommodate such use.

C.2.1 Length

Where foot or mile is writen in bold face in the following tables, it is to be understood as the survey foot or mile rather than international foot or mile.

		Gunter's chain measure *or* surveyor's chain measure	
1 inch	1 in		
1 foot = 12 inches	1 ft		
1 yard = 3 feet	1 yd		
		1 link	1 li
1 rod = 16.5 **feet**	1 rd	1 chain = 100 li	1 ch
1 furlong = 40 rods	1 fur		
1 survey mile = 8 furlongs	mi	1 survey mile = 80 ch	1 mi

C.2.2 Area

Area	
1 square inch	1 in^2
1 square foot = 144 sq in	1 ft^2
1 square yard = 9 sq ft	1 yd^2
1 square rod = 272.25 square **feet**	1 sq rd
1 acre = 160 sq rd	
1 square **mile** = 640 acres	1 mi^2
1 section of land = 1 square **mile**	
1 township = 36 sections	

C.2.3 Volume

Volume	
1 cubic inch	1 in^3
1 cubic foot = 1,728 in^3	1 ft^3
1 cubic yard = 27 ft^3	1 yd^3

Liquid measure		Dry measure	
1 minim [ˈmɪnɪm]			
1 fluid dram = 60 minims	1 fl dr		
1 fluid ounce = 8 fl dr	1 fl oz		
1 gill = 4 fl oz	1 gi		
1 pint = 4 gi	1 pt	1 pint = 33.60 in^3	1 pt
1 quart = 2 pt	1 qt	1 quart = 2 pt	1 qt
1 gallon = 4 qt = 231 in^3	1 gal		
		1 peck = 8 qt	1 pk
		1 bushel = 4 pk	1 bu

The words ‘fluid dram’ and ‘fluid ounce’ are also spelled — in the USA — ‘fluidram’ and ‘fluidounce’ respectively.

When necessary to distinguish the *liquid* pint or quart from the *dry* pint or quart, the word ‘liquid’ or the abbreviation ‘liq’ should be used in combination with the name or abbreviation of the *liquid* unit whereas the word ‘dry’ should be used in combination with the name or abbreviation of the *dry* unit.

Apothecaries’ fluid measure			
US		British	
1 minim	1 min	1 minim	1 min.
		1 fluid scruple = 20 min.	1 fl. scr.
1 fluid dram = 60 min	1 fl dr	1 fluid drachm = 3 fl. scr.	1 fl. dr.
1 fluid ounce = 8 fl dr	1 fl oz	1 fluid ounce = 8 fl. dr.	1 fl. oz
1 pint = 16 fl oz	1 pt	1 pint = 20 fl. oz	1 pt.
1 quart = 2 pt	1 qt		
1 gallon = 4 qt = 231 in^3	1 gal	1 gallon = 8 pt. = 277.42 cu. in.	1 gal.

C.2.4 Weight

hundredweight : **1** (in full **long hundredweight**) *Brit.* a unit of weight equal to 112 lb avoirdupois (about 50·8 kg). **2** (in full **metric hundredweight**) a unit of weight equal to 50 kg. **3** (in full **short hundredweight**) *US* a unit of weight equal to 100 lb (about 45·4 kg).

troy : (in full **troy weight**) a system of weights used for precious metals and gems, [2] with a pound of 12 ounces or 5,760 grains. [Middle English probably from TROYES]

The grain is the basis of all the weight systems and its value is always the same. One grain is equal to 64.798 91 mg.

Avoirdupois Weight	
1 grain	1 gr
1 dram	1 dr
1 ounce = 16 dr	1 oz
1 pound = 16 oz = 7,000 gr	1 lb
1 hundredweight = 100 lb	1 cwt
1 ton = 20 cwt	1 t
1 long *or* gross hundredweight = 112 lb	
1 long *or* gross ton = 2,240 lb	

Troy Weight		Apothecaries' Weight	
1 grain	1 gr	1 grain	1 gr
		1 scruple = 20 gr	1 s ap
1 pennyweight = 24 gr	1 dwt		
		1 dram = 3 s ap	1 dr ap
1 ounce troy = 20 dwt	1 oz t	1 ounce = 8 dr ap	1 oz ap
1 pound troy = 12 oz t	1 lb t	1 pound = 12 oz ap	1 lb ap

When necessary to distinguish the *avoirdupois* dram from the *apothecaries'* dram, or to distinguish the *avoirdupois* dram or ounce from the *fluid* dram or ounce, or to distinguish the *avoirdupois* ounce or pound from the *troy* or *apothecarie's* ounce or pound, the word 'avoirdupois' or the abbreviation 'avdp' should be used in combination with the name or abbreviation of the avoirdupois unit.

When the terms 'hundredweight' and 'ton' are used unmodified, they are commonly understood to mean the 100-pound hundredweight and the 2,000-pound ton respectively; these units may be designated [ˈdezɪgneɪtɪd] 'net' or 'short' when necessary.

[2] gem: [dʒeɪm] a precious stone esp. when cut and polished or engraved.

C.3 The SI System

C.3.1 Système Internationale d'Unités

[*Excerpt from The New York Public Library Science Desk Reference*]

The metric system was devised by scientists appointed by the French National Assembly during the French Revolution. The system was developed mainly as a standard to replace the numerous measurement systems in use throughout the country, but the system also stood for defiance against the previous governement's standard measurement system.

The first measurement, the meter [3] [ˈmiːtəʳ], was based on the circumference of the Earth measured on a line through Paris and the north and south poles [pəʊl]. The line was devided by 40, 000, 000, and each division was called a meter (from the Greek word *metron*, 'measure'). The standards of length were defined multiplying or dividing the meter by various factors of 10. Later, the meter was further defined as the length equal to 1, 650, 736.73 times the wavelength of orange light emitted when a gas consisting of a pure isotope [ˈaɪsəʊtəʊp] of krypton [ˈkrɪptɒn] (mass number 86) is excited in an electrical [ɪˈlektrɪkl] discharge [ˈdɪstʃɑːdʒ]. In 1983, the wavelength definition was replaced by the distance light travels in a vacuum in 1/299, 792, 458 second.

Other metric [ˈmetrɪk] measurements also have certain set standards. For example, the *gram* was originally set as the mass of 1 cubic centimeter of water under standard conditions. The modern kilogram [ˈkɪləʊgræm] (1, 000 grams) is equal to the mass of an international kilogram stored at Sèvres, France; a prototype [ˈprəʊtəʊtaɪp] is also located at the United States Bureau of Standards.

The metric system has been incorporated into the International System of Units (shortened to SI, from the French *Système International d'Unités*), which is now the standard measurement system for most fields of science. Within the SI system, the units are multiplied by factors of 10 when converting from one unit to another.

There are seven base units of the SI system: the *meter* (m, a measure of length), the *kilogram* (kg, a measure of weight), the *second* [ˈsekənd] (s, a measure of time), the *ampere* [ˈæmpeəʳ] (A, a measure of electric [ɪˈlektrɪk] current [ˈkʌrənt]), the *kelvin* [ˈkelvɪn] (K, a measure of temperature [ˈtemprətʃəʳ]), the *mole* [məʊl] (mol, a measure of the amount of a substance), and the *candela* [kænˈdelə] (cd, a measure of luminous [ˈluːmɪnəs] intensity), and two supplementary units, the *radian* [ˈreɪdjən] (rad, plane angles) and *steradian* [stəˈreɪdjən] (sr, solid angles). [*from* [21]]

[3] US spelling of 'metre'.

C.3.2 Metric Prefixes

Here are the number-prefixes recommended by the *Conférence générale des Poids et Mesures* in 1991.

Prefix	Abbr.	Factor	Prefix	Abbr.	Factor
deca-	da	10^1	deci-	d	10^{-1}
hecto-	h	10^2	centi-	c	10^{-2}
kilo-	k	10^3	milli-	m	10^{-3}
mega-	M	10^6	micro-	μ	10^{-6}
giga-	G	10^9	nano-	n	10^{-9}
tera-	T	10^{12}	pico-	p	10^{-12}
peta-	P	10^{15}	femto-	f	10^{-15}
exa-	E	10^{18}	a(t)to-	a	10^{-18}
zetta-	Z	10^{21}	zepto-	z	10^{-21}
yotta-	Y	10^{24}	yocto-	y	10^{-24}

C.3.3 Units of the SI

Derived units with special names					
Physical quantity	*Pronon.*	*Name*	*Pronon.*	*Symbol*	*Def.*
frequency	ˈfriːkwənsɪ	hertz	hɜːts	Hz	s^{-1}
force	fɔːs	newton	ˈnjuːtn	N	m kg s^{-1}
energy	ˈenədʒɪ	joule	dʒuːl	J	N m
power	paʊəʳ	watt	wɒt	W	J s^{-1}
pressure	ˈpreʃəʳ	pascal	ˈpæsk(ə)l	Pa	N m^{-2}
electric charge	tʃɑːdʒ	coulomb	ˈkuːlɒm	C	A s
electromotive force	ɪˌlektrəʊˈməʊtɪv	volt	vəʊlt	V	W A^{-1}
electic resistance	rɪˈzɪstəns	ohm	əʊm	Ω	V A^{-1}
electric conductance	kənˈdʌktəns	siemens	ˈsiːmənz	S	A V^{-1}
electric capacitance	kəˈpæsɪtəns	farad	ˈfærəd	F	C V^{-1}
magnetic flux	mægˈnetɪk flʌks	weber	ˈveɪbəʳ	Wb	V s
inductance	ɪnˈdʌktəns	henry	ˈhenrɪ	H	Wb A^{-1}
magnetic flux density	ˈdensɪtɪ	tesla	ˈteslə	T	Wb m^{-2}
luminous flux	ˈluːmɪnəs	lumen	ˈluːmɪn	lm	cd sr
illumination	ɪˌluːmɪˈneɪʃn	lux	lʌks	lx	lm m^{-2}

C.3.4 The names of the units

Ampère : André-Marie (1775–1836), French physicist, mathematician, and philosopher. He was a child prodigy who became one of the founders of electromagnetism and electrodynamics, and is best known for his analysis of the relationship between magnetic force and electric current. Ampère developed a precursor of the galvanometer.

Coulomb : Charles-Augustin de (1736–1806), French military engineer. He conducted research on structural mechanics, elasticity, friction, electricity, and magnetism. He is best known for his Coulomb's Law, established with a sensitive torsion balance in 1785, according to which the forces between two electrical charges are proportional to the product of the sizes of the charges and inversely proportional to the square of the distance between them. Coulomb's verification of the inverse square law of electrostatic force enabled the quantity of electric charge to be defined.

Faraday : [ˈfærəˌdeɪ] Michael (1791–1867), English physicist and chemist. One of the greatest experimentalists, he was largely self-educated. Appointed by Sir Humphry Davy as his assistant at the Royal Institution, he initially concentrated on analytical chemistry and discovered benzene in 1825. His most important work was in electromagnetism, in which field he demonstrated electromagnetic rotation and discovered electromagnetic induction (the key to the development of the electric dynamo and motor). His concept of magnetic lines of force formed the basis of the classical field theory of electromagnetic behaviour. He also discovered the laws of electrolysis.

Henry : Joseph (1797–1878), American physicist, born in Albany [ˈɔːlbənɪ], New York. In 1832 he became professor of natural philosophy at Princeton, and in 1846 first secretary of the Smithsonian Institution. He discovered electrical induction independently of Michael Faraday, constructed the first electromagnetic motor (1829), demonstrated the oscillatory nature of electric discharge (1842), and introduced a system of weather forecasting.

Hertz : Heinrich Rudolf (1857–1894), German physicist and pioneer of radio communication. He worked for a time as Helmholtz's assistant in Berlin, and in 1886 began studying the electromagnetic waves that Maxwell had predicted. He demonstrated them experimentally, and also showed that they behaved like light and radiant heat, thus proving that these phenomena, too, were electromagnetic. In 1889 he was appointed professor of physics at Bonn, but he died of blood-poisoning at the early age of 37.

Joule : James Prescott (1818–1889), English physicist. Experimenting in his private laboratory and at the family's brewery, he established that all forms of energy were basically the same and interchangeable — the basic principle of what is now called the first law of thermodynamics. Among other things, he measured the thermal effects of an electric current due to resistance of the wire, establishing the law governing this. In 1852 he and William Thomson, later Lord Kelvin, discovered the fall in temperature when gases expand (the Joule-Thomson effect), which led to the development of the refrigerator and to the science of cryogenics.

Kelvin : William Thomson, 1st Baron (1824–1907), British physicist, professor of natural philosophy at Glasgow from 1846 to 1895. He restated the second law of thermodynamics in 1850, and introduce the absolute scale of temperature. His concept of an electromagnetic field influenced Maxwell's electromagnetic theory of light, which Kelvin never accepted. He was involved in the laying of the first Atlantic cable, for which he invented several instruments, and he devised many scientific instruments for other purposes.

Newton : Sir Isaac [ˈaɪzək] (1642–1727), English mathematician and physicist, the greatest single influence on theoretical physics until Einstein. His most productive period was 1665–1667, when he retreated temporarily from Cambridge to his isolated home in Lincolnshire during the Great Plague. He discovered the binomial theorem, and made several other contributions to mathematics, notably differential calculus and its relationship with integration. A bitter quarrel with Leibnitz ensued as to which of them had discovered calculus first. In his major treatise, *Principia Mathematica* (1687), he gave a mathematical description of the laws of mechanics and gravitation, and applied these to planetary and lunar motion. For most purposes Newtonian mechanics has survived even the introduction of relativity theory and quantum mechanics, to both of which it stands as a good approximation. Another influential work was *Opticks* (1704), which gave an account of his optical experiments and theories, including the discovery that white light is made up of a mixture of colours. In 1699 Newton was appointed Master of the Mint; he entered Parliament as MP for Cambridge University in 1701, and in 1703 was elected president of the Royal Society.

Ohm : Georg Simon (1789–1854), German physicist. He published two major papers in 1826, which between them contained the law that is named after him. This states that the electric current flowing in a conductor is directly proportional to the potential difference (voltage), and inversely proportional to the resistance. Applying this to a wire of known diameter and conductivity, the current is inversely proportional to length.

Pascal : Blaise (1623–1662), French mathematician, physicist, and religious philosopher. A child prodigy, before the age of 16 he had proved an important theorem in the projective geometry of conics, and at 19 constructed the first mechanical calculator to be offered for sale. He discovered that air has weight, confirmed that the vacuum could exist, and derived the principle that the pressure of a fluid at rest is transmitted equally in all directions. He also founded the theory of probabilities, and developed a forerunner of integral calculus. He later entered a Jansenist convent, where he wrote two classics of French devotional thought, the *Lettres Provinciales* (1656–7), directed against the casuistry of the Jesuits, and *Pensées* (1670), a defence of Christianity.

Siemens : Ernst Werner von (1816–1892), German electrical engineer. He developed electroplating and an electric generator which used an electromagnet, and set up a factory which manufactured telegraph systems and electric cables and pioneered electrical traction. His brother Karl Wilhelm (Sir Charles William, 1823–1883) moved to England, where he developed the open-hearth furnace and designed the cable-laying steamship *Faraday*, and also designed the electric railway at Portrush in Northern Ireland. A third brother Friedrich (1826–1904) worked both for Werner in Germany and with Charles in England; he applied the principles of the open-hearth furnace to glassmaking.

Tesla : Nikola (1856–1943), American electrical engineer and inventor, born in what is now Croatia of Serbian descent. He emigrated to the US in 1884 and worked briefly on motors and direct-current generators with Thomas Edison before joining the Westinghouse company, where he developed the first alternating-current induction motor (1888) and made contribution to long-distance electrical power transmission. Tesla also studied high-frequency current, devloping several forms of oscillators and the tesla coil, and developed wireless guidance system for ships. Although his inventions revolutionized the electrical industry, he died in poverty.

Volta : Alessandro Giuseppe Antonio Anastasio, Count (1745–1827), Italian physicist. He was the inventor of a number of important electrical instruments, including the electrophorus and the condensing electroscope, but is best known for the voltaic pile or electrochemical battery (1800) — the first device to produce continuous electric current. The impetus for this was Luigi Galvani's claim to have discovered a new kind of electricity produced in animal tissue, which Volta ascribed to normal electricity produced by the contact of two dissimilar metals.

Watt : James (1736–1819), Scottish engineer. He greatly improved the efficiency of the Newcomen beam engine by condensing the spent steam in a separate chamber, allowing the cylinder to remain hot. The improved engines were adopted for a variety of purposes, especially after Watt entered into a business partnership with the engineer Matthew Boulton. Watt continued inventing until the end of his life, introducing rotatory engines, controlled by a centrifugal governor, and devising a chemical method of copying documents. He also introduced the term *horsepower*.[4]

Weber : Wilhelm Eduard (1804–1891), German physicist. His early researches were in acoustics and animal locomotion, but he is chiefly remembered for his contributions in the fields of electricity and magnetism. He proposed a unified system for electrical units, determined the ratio between the units of electrostatic and electromagnetic charge, and devised a law of electrical force (later replaced by Maxwell's field theory). He went on to investigate electrodynamics and the nature and role of electric charge.

C.4 History and Future of the Imperial System

C.4.1 History of the Imperial units

[from `www.unc.edu/~rowlett/units`;
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Distance

In all traditional measuring systems, short distance units are based on the dimensions of the human body. The inch represents the width of a thumb; in fact, in many languages, the word for 'inch' is also the word for 'thumb'. The foot (12 inches) was originally the length of a human foot, although it has evolved to be longer than most people's feet. The yard (3 feet) seems to have gotten its start in England as the name of a 3-foot measuring stick, but it is also understood to be the distance from the tip of the nose to the end of the middle finger of the outstretched hand. Finally, if you stretch your arms out to the sides as far as possible, your total 'arm span', from one fingertip to the other, is a fathom (6 feet).

Historically, there are many other 'natural units' of the same kind, including the digit (the width of a finger, 0.75 inch), the nail (length of the last two joints of the middle finger, 3 digits or 2.25 inches), the palm (width of the palm, 3 inches), the hand (4 inches), the shaftment (width of the hand and outstretched thumb, 2 palms or 6 inches), the span

[4] [ˈhɔːsˌpaʊəʳ] an imperial unit of power (symbol **hp**) equal to 550 foot-pounds per second (about 750 W).

(width of the outstretched hand, from the tip of the thumb to the tip of the little finger, 3 palms or 9 inches), and the cubit (length of the forearm, 18 inches).

In Anglo-Saxon England (before the Norman conquest of 1066), short distances seem to have been measured in several ways. The inch (*ynce*) was defined to be the length of 3 barleycorns, which is very close to its modern length. The shaftment was frequently used, but it was roughly 6.5 inches long. Several foot units were in use, including a foot equal to 12 inches, a foot equal to 2 shaftments (13 inches), and the 'natural foot' (*pes naturalis*, an actual foot length, about 9.8 inches). The fathom was also used, but it did not have a definite relationship to the other units.

When the Normans arrived, they brought back to England the Roman tradition of a 12-inch foot. Although no single document on the subject can be found, it appears that during the reign of Henry I (1100–1135) the 12-inch foot became official, and the royal government took steps to make this foot length known. A 12-inch foot was inscribed on the base of a column of St. Paul's Church in London, and measurements in this unit were said to be 'by the foot of St. Paul's' (*de pedibus Sancti Pauli*). Henry I also appears to have ordered construction of 3-foot standards, which were called 'yards,' thus establishing that unit for the first time in England. William of Malmsebury [ˈmɑːzbərɪ] wrote that the yard was 'the measure of his [the king's] own arm,' thus launching the story that the yard was defined to be the distance from the nose to the fingertip of Henry I. In fact, both the foot and the yard were established on the basis of the Saxon *ynce*, the foot being 36 barleycorns and the yard 108.

Meanwhile, all land in England was traditionally measured by the *gyrd* or rod, an old Saxon unit probably equal to 20 'natural feet'. The Norman kings had no interest in changing the length of the rod, since the accuracy of deeds and other land records depended on that unit. Accordingly, the length of the rod was fixed at 5.5 yards (16.5 feet). This was not very convenient, but 5.5 yards happened to be the length of the rod as measured by the 12-inch foot, so nothing could be done about it. In the Saxon land-measuring system, 40 rods make a furlong (*fuhrlang*), the length of the traditional furrow (*fuhr*) as ploughed by ox teams on Saxon farms. These ancient Saxon units, the rod and the furlong, have come down to us today with essentially no change.

Longer distances in England are traditionally measured in miles. The mile is a Roman unit, originally defined to be the length of 1000 paces of a Roman legion. A 'pace' here means two steps, right and left, or about 5 feet, so the mile is a unit of roughly 5000 feet. For a long time no one felt any need to be to precise about this, because distances longer than a furlong did not need to be measured exactly. It just didn't make much difference whether the next town was 21 or 22 miles away. In medieval England, various mile units seem to have been used. Eventually, what made the most sense to people was that a mile should equal 8 furlongs, since the furlong was an English unit roughly equivalent to the Roman stadium and the Romans had set their mile equal to 8 tadia. This correspondence is not exact: the furlong is 660 English feet and the stadium is only 625 slightly-shorter Roman feet.

In 1592, Parliament settled this question by setting the length of the mile at 8 furlongs, which works out to 1760 yards or 5280 feet. This decision completed the English distance system. Since this was just before the settling of the American colonies, British and American distance units have always been the same.

Reorganisations

Out of the welter[5] of medieval [ˌmedɪˈiːvl] weights and measures emerged several national systems, reformed and reorganized from time to time; ultimately nearly all these were replaced by metric. In Britain and its American colonies, however, the ancient system survived.

By the time of Magna Carta[6] (1215), abuses of weights and measures were so common that a clause was inserted in the charter to correct those on grain and wine. A few years later a royal ordinance [ˈɔːdɪnəns] entitled 'Assize [əˈsaɪz] of Weights and Measures' defined a broad list of units and standards [ˈstændəd] so successfully that it remained in force nearly 600 years. [. . .]

The sets of standards, which were sent out from London to the provincial towns, were usually of bronze or brass. Discrepancies[7] [dɪsˈkrepənsɪ] somehow crept into the system, and in 1496, following a Parliamentary inquiry, new standards were made and sent out, a procedure repeated in 1588, under Elizabeth I[8].

In 1592, Parliament settled the question of the mile by setting the length of the mile at 8 furlongs, which works out to 1760 yards or 5280 feet. This decision completed the English distance system. Since this was just before the settling of the American colonies, British and American distance units have always been the same.

No revision of law was found necessary for 200 years after Elizabeth's time, but several refinements and redefinitions were added. E. Gunter[9], a 17th-century mathematician, conceived the idea of taking the acre's breadth (4 perches [pɜːtʃ], or 22 yards), calling it a chain, and dividing it into 100 links.

The Act of 1878 redefined the yard: 'the straight line or distance between the centres of two gold plugs or pins in the bronze bar [. . .] measured when the bar is at the temperature of sixty-two degrees of Fahrenheit's thermometer, and when it is supported by bronze rollers placed under it in such a manner as best to avoid flexure of the bar.'

From 1893 until 1959, the yard was defined as equalling exactly 3600/3937 meter. In 1959 a small change was made in the definition of the yard to resolve discrepancies. Since then the yard is defined as being equal exactly to 0.9144 meter. At the same time it was decided that any data expressed in feet derived from geodetic surveys within the USA would continue to bear the relationship as defined in 1893. This foot is called the US survey foot, while the foot defined in 1959 is called the international foot.

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[5] [ˈweltəʳ]: a state of general confusion; (followed by *of*) a disorderly mixture or contrast.

[6] Magna Carta [ˌmægnəˈkætə] *also* Magna Charta [ˈtʃɑːtə]: the English political charter which King John was forced to sign by his rebellious barons at Runnymede [ˈrʌnɪˌmiːd] — a meadow on the south bank of the Thames at Egham [ˈegəm] near Windsor, Surrey. The barons were led by Archbishop Langton to frame a charter which effectively redefined the limits of royal power.

[7] discrepancy: difference; failure to correspond; inconsistency

[8] (1533–1603), daughter of Henri VIII, queen of England and Ireland 1558–1603.

[9] Edmund Gunter [ˈedmənd ˈgʌntə] 1581–1626.

Area

In all the English-speaking countries, land is traditionally measured by the acre, a very old Saxon unit which is either historic or archaic, depending on your point of view. There are references to the acre at least as early as the year 732. The word 'acre' also meant 'field', and as a unit an acre was originally a field of a size that a farmer could plough in a single day. In practice, this meant a field that could be ploughed in a morning, since the oxen had to be rested in the afternoon. Most area units were eventually defined to be the area of a square having sides equal to some simple multiple of a distance unit, like the square yard. But the acre was never visualised as a square. An acre is the area of a long and narrow Anglo-Saxon farm field, one furlong (40 rods) in length but only 4 rods wide. This works out, very awkwardly indeed, to be exactly $43,560$ square feet . If we line up 10 of these 4×40 standard acres side by side, we get 10 acres in a square furlong, and since the mile is 8 furlongs there are exactly $10 \times 8 \times 8 = 640$ acres in a square mile.

Weight

The basic traditional unit of weight, the pound, originated as a Roman unit and was used throughout the Roman Empire. The Roman pound was divided into 12 ounces, but many European merchants preferred to use a larger pound of 16 ounces, since a 16-ounce pound is conveniently divided into halves, quarters, or eighths. During the Middle Ages there were many different pound standards in use, some of 12 ounces and some of 16. The use of these weight units naturally followed trade routes, since merchants trading along a certain route had to be familiar with the units used at both ends of the trip.

In traditional English law the various pound weights are related by stating all of them as multiples of the grain, which was originally the weight of a single barleycorn. Thus barleycorns are at the origin of both weight and distance units in the English system.

The oldest English weight system has been used since the time of the Saxon kings. It is based on the 12-ounce troy pound, which provided the basis on which coins were minted and gold and silver were weighed. Since Roman coins were still in circulation in Saxon times, the troy system was designed to model the Roman system directly. The troy pound weighs 5760 grains, and the ounces weigh 480 grains, which is the traditional weight of the silver coin called the shilling. The shilling was equal to 20 pence (pennies), and therefore a pennyweight is $480/20 = 24$ grains. The troy system continued to be used by jewellers and also by druggists until the nineteenth century. Even today gold and silver prices are quoted by the troy ounce in financial markets everywhere.

Since the troy pound was smaller than the commercial pound units used in most of Europe, medieval English merchants often used a larger pound called the 'mercantile' pound (*libra mercatoria*). This unit contained 15 troy ounces, so it weighed 7200 grains. This unit seemed about the right size to merchants, but its division into 15 parts, rather than 12 or 16, was very inconvenient. Around 1300 the mercantile pound was replaced in English commerce by the 16-ounce avoirdupois pound. This is the pound unit still in common use in the USA and UK. Modelled on a common Italian pound unit of the late thirteenth century, the avoirdupois pound weighs exactly 7000 grains. The avoirdupois ounce, 1/16 pound, is divided further into 16 drachms. Unfortunately, the two English ounce units don't agree: the avoirdupois ounce is $7000/16 = 437.5$ grains while the troy ounce is $5760/12 = 480$ grains. Conversion between troy and avoirdupois units is so awkward, no one wanted to do it. The troy system quickly became highly specialised, used only for precious metals and for pharmaceuticals, while the avoirdupois pound was used for everything else.

Since at least 1400 a standard weight unit in Britain has been the hundredweight, which is equal to 112 avoirdupois pounds rather than 100. There were very good reasons for the odd size of this 'hundred': 112 pounds made the hundredweight equivalent for most purposes with competing units of other countries, especially the German *Zentner* and the French *quintal*. Furthermore, 112 is a multiple of 16, so the British hundredweight can be divided conveniently into 4 quarters of 28 pounds, 8 stone of 14 pounds, or 16 cloves of 7 pounds each. The ton, originally a unit of wine measure, was defined to equal 20 hundredweight or 2240 pounds.

During the nineteenth century, an unfortunate disagreement arose between British and Americans concerning the larger weight units. Americans, not very impressed with the history of the British units, redefined the hundredweight to equal exactly 100 pounds. The definition of the ton as 20 hundredweight made the disagreement carry over to the size of the ton: the British 'long' ton remained at 2240 pounds while the American 'short' ton became exactly 2000 pounds. (The American hundredweight became so popular in commerce that British merchants decided they needed a name for it; they called it the cental.) Today, most international shipments are reckoned in metric tons, which, coincidentally, are rather close in weight to the British long ton.

Volume

The names of the traditional volume units are the names of standard containers. Until the eighteenth century, it was very difficult to measure the capacity of a container accurately in cubic units, so the standard containers were defined by specifying the weight of a particular substance, such as wheat or beer, which they could carry. Thus the gallon, the basic English unit of volume, was originally the volume of eight pounds of wheat. This custom led to a multiplicity of units, as different commodities were carried in containers of slightly different sizes.

Gallons are always divided into 4 quarts, which are further divided into 2 pints each. For larger volumes of dry commodities, there are 2 gallons in a peck and 4 pecks in a bushel. Larger volumes of liquids were carried in barrels, hogsheads, or other containers whose size in gallons tended to vary with the commodity, with wine units being different from beer and ale units or units for other liquids.

The situation was still confused during the American colonial period, so the Americans were actually simplifying things by selecting just two of the many possible gallons. These two were the gallons that had become most common in British commerce by around 1700. For dry commodities, the Americans were familiar with the 'Winchester bushel,' defined by Parliament in 1696 to be the volume of a cylindrical container 18.5 inches in diameter and 8 inches deep. The corresponding gallon, 1/8 of this bushel, is usually called the 'corn gallon' in England. This corn gallon holds 268.8 cubic inches.

For liquids Americans preferred to use the traditional British wine gallon, which Parliament defined to equal exactly 231 cubic inches in 1707. As a result, the US volume system includes both 'dry' and 'liquid' units, with the dry units being about 1/6 larger than the corresponding liquid units.

In 1824, the British Parliament abolished all the traditional gallons and established a new system based on the 'Imperial' gallon of 277.42 cubic inches. The Imperial gallon was designed to hold exactly 10 pounds of water under certain specified conditions. Unfortunately, Americans were not inclined to adopt this new, larger gallon, so the traditional English 'system' actually includes three different volume measurement systems: US liquid, US dry, and British Imperial.

On both sides of the Atlantic, smaller volumes of liquid are traditionally measured in fluid ounces, which are roughly equal to the volume of one ounce of water. To accomplish this in the different systems, the smaller US pint is divided into 16 fluid ounces, and the larger British pint is divided into 20 fluid ounces.

C.4.2 Metrication in the UK

[from `www.metric.org.uk`; UK Metrication Association]

Parliament first debated the metric system on 13th April 1790. This was when parliamentarian Sir John Riggs Miller [Britain] and the Bishop of Autum, Prince Talleyrand [France] put to the British Parliament and French Assembly respectively, the proposition that the two countries should cooperate to equalise their weights and measures, by the joint introduction of the metric system.

There was no immediate progress although there were many positive debates in the second half of the 19th Century. For example, 1st July 1863 the Bill for a compulsory change to the metric system was approved by 110 votes to 75 votes.

The following year, 9th March 1864, the House of Lords debated a Bill to permit the use of metric weights and measures in trade. One supporter noted that Englishmen were notorious for liking old terms and old habits and he hoped that the new nomenclature would not be diverted by attempts at ridicule. Parliament passed the Bill and this became the Metric Weights and Measures Act 1864. On the 24th February 1868 a parliamentary proposal to set Imperial cut-off dates was withdrawn on promise of a Royal Commission of enquiry. The Enquiry Report was positive, and on the 26th July 1871 Britain almost became a metric country. The government lost the Bill to make metric compulsory after two years, by only 82 votes to 77 votes. An argument that might have influenced opponents was a plea that Britain would be 'letting down America and our colonies' who had harmonised their systems with the ones in use in Britain[10].

There were further debates, and near misses, in the UK Parliament in 1872 and 1896, before a comprehensive debate [21st June - 6th August 1897] concluded by legalising the use of metric for all purposes. There were no contrary votes.

Metrication continued to be debated for the next 10 years. In 1904 The House of Lords unanimously voted to make metric compulsory after two years. It was claimed that the Austrian and German nations had successfully made metric compulsory with a changeover time of only 'one week'! The Government said they would not obstruct the proposal, but the Bill was never adopted in the Commons. Two similar debates in 1907 failed. Conflicts in Europe put further political consideration of metrication out of mind until the publication of a Government White Paper on Weights and Measures 10th May 1951. The 1951 White Paper was in fact the 28th Report put to Parliament during the preceeding 100 years. This latest report was in response to the the Hodgson Committee Report published in 1949. Eventually we had the Weights and Measures Act 1963; a long

[10]At that time the American Congress had emulated Britain by allowing contracts in metric. A particularly strong USA advocate for metric was John Quincy Adams.

series of Parliamentary questions to Ministers and the Federation of British Industries [now the CBI[11]*] lobby in favour of metrication in 1965. These initiatives culminated with the creation of the Metrication Board in 1969 by Anthony Wedgewood Benn, Minister of Technology. The target date for completion was end 1975. The transition to metrication and the role of the Board were given positive support and encouragement by Geoffrey Howe the responsible Minister of the new Government in 1972. Indeed at that time, and until circa 1977-1978, there was good, sensible and steady progress.*

Prepackaged food changed but the really difficult issue to emerge affected retailers of 'loose weight' products. The retail problem was that metric prices would always appear to be more expensive than their nearest Imperial equivalent.

The product which brought all voluntary retail initiatives to a full stop was the experience of the floor covering and carpet retailers. Their 1975 change to sales by the sq. metre started well, but in 1977 one of the major High Street retailers found enormous commercial advantage in reverting to sales by the square yard. Consumers could not be persuaded to believe that goods costing, for example, £10 per square yard or £12 per square metre were virtually priced the same. Consumers bought, in very significant volume, the apparently cheaper priced imperial version. Metrication of carpet sales entered into full scale reverse and the Chambers of Trade and retail associations pressed for firm Government leadership, i.e. *compulsory cut-off. Then the political nerve began to fail.*

The necessary Order, drafted by the Board of Trade in 1978, was agreed by a huge range of retail trade, industry, engineering, consumer, trade union, elderly person, sporting and educational organisations and . . . the overwhelming number of parliamentarians. A small number of critics, in each political party, did voice opposition to the element of compulsion but this seemed to come from a relatively small minority within the Eurosceptic movement.

However, the initiative was in the hands of Secretary of State for Trade, Roy Hattersley and a General Election was expected in 1979. Labour lost the election and Margaret Thatcher became Prime Minister.

One Conservative backbencher, Sally Oppenheim had been almost the lone but persistent critic of the metric programme. Ironically she was appointed junior Minister of Consumer Affairs at the DTI and then metrication was added to her portfolio. In letters to MP's and associations she made it clear [a] she was not opposed the metrication in principle, [b] metrication was not the result of Britain's accession to the EEC but [c] she did object to measures which would compel people to adopt metric against their will. Proponents of metrication, trade and consumer organisations, officials and the Metrication Board explained and argued that a voluntary change at retail level was absolutely impossible . . . it could never happen. It was a recipe for confusion, waste and duplication. Government had to lead over the last hurdle. It did, it led backwards. In 1980 the Metrication Board was abolished.

In truth the Metrication Board had little else to do. Every possible programme had been agreed, consumer information campaigns composed and there was nothing to do until or unless a date was fixed for the completion of the transition. We little knew then the die was set for a further 20 years of waste, confusion and argument. Successive DTI Ministers did nothing to inform consumers or public opinion. They did nothing to refute the new 'big lie' namely, that Britain was being forced to change because of the European Commission. In fact, during the past 20 years most Commission Officials, European Politicians and

[11] Confederation of British Industry

businesses in Continental Europe 'couldn't have given a damn' whether Britain changed to the metric system or not. They seemed to quite like the idea of Britain shooting itself in its economic foot, by imposing upon itself the extra costs and waste of maintaining a dual system. For twenty years not one single British Minister has attempted to explain the advantages of metrication. Most tried to pretend or imply they were protecting our British culture from the European bully.

Jim HUMBLE OBE[12], Director of the Metrication Board (1978–1980)

However, the government had already signed up to the European Directive 71/354 to harmonise our units of measurement by requiring SI. This resulted in several further Directives which set out transition dates for phasing out most non-metric units. UK legislation was then amended to enact these changes. 1995 saw the removal of the pound (weight) and pint for labelling pre-packed goods.

Perhaps the most significant change took place at the end of 1999; as of January 1st 2000 it has no longer been legal to sell loose products (vegetables, fruit, cheese, meat, nails, ground coffee, etc.) by reference to the ounce, pound, pint or gallon (with the exception of draught beer). Pints of beer are usually spoken about in the same breath as the pint of milk in returnable containers, but there is one significant difference, in that it is now perfectly legal to sell milk in metric sizes.

Although a great deal has happened in the UK over the past 30 years, there is still a lot of ignorance of the current situation. Some people think it's already gone too far, others don't seem to be aware of what has actually happened. One school of thought is that SI (metric) is OK for some fields, e.g. scientific and engineering endeavours, but it is not suitable for others. For example, as Britain is an island, and the design of our road signs doesn't affect how we do trade with the rest of the world, it is not necessary for road signs to convert to metric. However, SI is a coherent system of units, and including non-metric units defeats the point. We buy our petrol by the litre, so it's easier to calculate consumption using kilometres; coaches and trucks have odometers recording kilometres, and speedometers showing km/h (not kph, please!) in prominent form. The argument also ignores the fact that the UK shares a border with Ireland, which has largely converted its distance signs, and is planning to change its speed limit signs in 2001.

[12]Officer of the Order of the British Empire

Appendix D

Vocabulary

Remark: *A dag (†) shows at the end of a line if an example or an explanation is provided about the English word given in the line. The reader will find the example or explanation after the current table.*

School		
school subject	skuːl ˈsʌbdʒɪkt	matière scolaire
curriculum plur. *-la*	kəˈrɪkjələm -lə	programme †
syllabus plur. *-bi*	ˈsɪləbəs -ˌbaɪ	programme †

curriculum : the subjects that are studied or prescribed for study in a school. [21]

stream : (*Brit.*) a group of shoolchildren taught together as being of similar ability for a given age. [20]

syllabus : **1** the programme or outline of a course study, teaching, etc. **2** the statement of the requirements for a particular examination. [21]

For example: 'To use this book most effectively you need to know which Study Units are in your course, and which are not. Therefore you need to identify the syllabus you are studying.' p. vi [11]

School stationery		
stationery	ˈsteɪʃənərɪ	fournitures †
school-bag	ˈskuːlbæg	sac d'école
tracing paper	ˈtreɪsɪŋˌpeɪpə^{r}	papier calque
cardboard	ˈkɑːdbɔːd	carton

School stationery — continued		
textbook	ˈteks(t)bʊk	manuel
exercise book	ˈeksəsaɪzˌbʊk	cahier de devoirs
copybook	ˈkɒpɪbʊk	cahier
notebook	ˈnəʊtbʊk	bloc-note, cahier, carnet
writing pad	ˈraɪtɪŋpæd	bloc-note, cahier, carnet
jotter	ˈdʒɒtəʳ	cahier de brouillon
binder	ˈbaɪndəʳ	classeur
sheet	ʃiːt	feuille
folder	ˈfəʊldəʳ	chemise
pencil	ˈpensl	crayon
pencil-case	ˈpenslˌkeɪs	trousse
pencil-sharpener	ˈpenslˌsɑːpnəʳ	taille-crayon
fountain pen	ˈfaʊntɪnpen	stylo plume
ballpoint (pen)	ˈbɔːlpɔɪnt (pen)	stylo (à bille)
crayon	ˈkreɪən	crayon de couleur
box of colors	bɒks əv ˈkʌləz	boite de peinture
biro	ˈbaɪ(ə)rəʊ	bic
felt-tip pen	ˌfeltɪpˈpen	feutre
eraser	ɪˈreɪzəʳ	effaceur, gomme
rubber	ˈrʌbəʳ	gomme
ruler	ruːləʳ	règle
(a pair of) compasses	(ə peər əv) ˈkʌmpəsɪz	compas
(set) square	(set) skweəʳ	équerre
(pocket) calculator	ˈkælkjʊleɪtəʳ	calculatrice
glue or *paste*	gluː / peɪst	colle
paperclip	ˈpeɪpəklɪp	trombone
protractor	prəˈtræktəʳ	rapporteur

Stationer : a person who sells writing materials etc. [20]
Stationery : writing materials etc. sold by a stationer. [20]

The ISO [1] specifies an 'A' size series [ˈsɪəriːz] of drawing sheets for technical drawing. The basic A0 sheet has an area of 1 square metre. Other A

[1] International Organization for Standardization

size sheets are A1, A2, A3, A4 and A5. All A size sheets have their edge lengths in the same proportion. [35]

Papers are commonly measured by their weight — known as so many grams per square metre — **sgm**. . . . Papers suitable for technical drawings are: **Cartridge paper** – a good quality paper for pencil drawings . . . ; **Detail paper** – a lighter paper for pencil and colour work . . . ; **Grid papers** . . . Square, isometric and perspective grids. **Tracing paper**; **Papers** and **boards** for **ink work** . . . [35]

Erasers : essential for correcting mistakes. Vinyl [ˈvaɪnɪl] erasers are preferable to rubber erasers – they make a cleaner job of 'rubbing out'. Be careful of rubber dust formed when erasing from pencil drawing. It can be a source of annoyance causing smudges [ˈsmʌdʒɪz] (tache, bavure) on your drawing if it is allowed to accumulate unnecessarily. [35]

Pencils : can be purchased in nine grades of 'hardness' — from H to 9H — and six grades of 'blackness' — from B to 6 B. There are also two other grades — F and HB . . . Many draughtsmen like to sharpen their 2H pencils to a 'chisel' point and their HB pencils to a round point. [35]

Punctuation marks		
punctuation mark	ˌpʌŋktʃʊˈeɪʃn mɑːk	signe de ponctuation
full stop	ˌfʊlˈstɒp	point
period (US)	ˈpɪərɪəd	point
comma	ˈkɒmə	virgule
dash	dæʃ	tiret —
apostrophe	əˈpɒstrəfɪ	apostrophe
exclamation mark	eksklәˈmeɪʃnˌmɑːk	point d'exclamation
question mark	ˈkwestʃənˌmɑːk	point d'interrogation
colon	ˈkəʊlən	deux points :
semicolon	ˌsemɪˈkəʊlən	point-virgule
dot	dɒt	point décimal†
inverted commas	ɪnˈvɜːtɪd —	' '
solidus -di	ˈsɒlɪdəs -ˌdaɪ	barre oblique /
slash	slæʃ	barre oblique
ellipsis	ɪˈlɪpsɪs	points de suspension ...
hyphen	ˈhaɪfən	trait d'union
ampersand	ˈæmpəˌsænd	esperluète &

Dot the i's and cross the t's: (*colloquial*) **1** be minutely accurate, emphasize details. **2** add the final touches to a task, exercise, etc.

Dotted line : a line of dots on a document, especially to show a place left for a signature.

The inverted commas are also known as *quotation marks* or *quotes* which is an abbreviation of the preceding words. Usually in British English quotations are given thus: 'Why does he use the word "déjà vu"?'. The more common quotation marks are the single ones. In the USA, we usually find: "Why does he use the word 'déjà vu'?".

Sciences		
science	ˈsaɪəns	science
natural science	ˈnætʃr(ə)l	sciences naturelles
physics	ˈfɪzɪks	physique
physicist	ˈfɪzɪsɪst	physicien
mathematics	ˌmæθˈmætɪks	mathématiques
mathematician	ˌmæθməˈtɪʃn	mathématicien
biology	baɪˈɒlədʒɪ	biologie
chemistry	ˈkemɪstrɪ	chimie
geography	dʒɪˈɒgrəfɪ	géographie
geology	dʒɪˈɒlədʒɪ	géologie

Reasoning and reckoning		
reasoning	ˈriːzənɪŋ	raisonnement
definition	ˌdefɪˈnɪʃn	définition
axiom	ˈæksɪəm	axiome
to assume	əˈsjuːm	supposer
assumption	əˈsʌmʃn	supposition
to conjecture	kənˈdʒektʃəʳ	conjecturer †
hypothesis plur. *-ses*	haɪˈpɒθɪsɪs -siːz	hypothèse
conclude	kənˈkluːd	conclure
conclusion	kənˈkluːʒn	conclusion
condition	kənˈdɪʃn	condition
conditional	kənˈdɪʃnl	conditionnel
consequence	ˈkɒnsɪkwəns	consequence
converse	ˈkɒnvɜːs	réciproque (*nom*)

Reasoning and reckoning — continued		
conversely	ˌkɒnˈvɜːslɪ	réciproquement
to imply	ɪmˈplaɪ	impliquer
implication	ˌɪmplɪˈkeɪʃn	implication
then	ðen	alors
therefore or *therefor*	ˈðeəfɔːʳ	donc
for	fɔːʳ	car (*conj. coord.*)
if	ɪf	si
if and only if		si et seulement si
equivalent	ɪˈkwɪvələnt	équivalent
equivalence	ɪˈkwɪvələns	équivalence
statement	ˈsteɪtmənt	énoncé, affirmation
to state	steɪt	affirmer, énoncer
proposition	ˌprɒpəˈzɪʃn	proposition
theorem	ˈθɪərəm	théorème
corollary	kəˈrɒlərɪ	corollaire
proof	pruːf	preuve, démonstration
to prove	pruːv	démontrer
to demonstrate	ˈdemənstreɪt	démontrer
to deduce	dɪˈdjuːs	déduire
hint	hɪnt	indication
truth	truːθ	vérité
true	truː	vrai
false	fɔːls	faux
to determine	dɪˈtɜːmɪn	déterminer
to calculate	ˈkælkjʊleɪt	calculer
calculation	ˌkælkjʊˈleɪʃn	calcul
to reckon	ˈrekən	compter, calculer
reckoning	ˈrekənɪŋ	compte, calcul
to classify	ˈklæsɪfaɪ	classer
calculator	ˈkælkjʊleɪtəʳ	calculatrice
key	kiː	touche
memory	ˈmemərɪ	mémoire
to store	stɔːʳ	mettre en mémoire
programmable	prəʊˈgræmbl	programmable
computer	kəmˈpjuːtəʳ	ordinateur

Reasoning and reckoning — continued		
algorithm	ˈælgərɪðəm	algorithme
program	ˈprəʊgræm	programme
instruction	ɪnˈstrʌkʃn	instruction
step		pas (de programme)
jump	dʒʌmp	saut
loop	luːp	boucle
input	ˈɪnpʊt	entrée
output	ˈaʊtpʊt	sortie
display	dɪˈspleɪ	affichage
to display	dɪˈspleɪ	afficher
bit	bɪt	bit
byte	baɪt	octet
word	wɜːd	mot
flowchart	ˈfləʊˌtʃɑːt	ordinogramme
software	ˈsɒftweəʳ	logiciel
abacus	ˈæbəkəs	boulier, abaque
database	ˈdeɪtəbeɪs	base de données
spreadsheet	ˈspredʃiːt	tableur
wordprocessor	ˈwɜːdˌprəʊsesəʳ	traitement de texte
to estimate	ˈestɪmeɪt	estimer
estimation	ˌestɪˈmeɪʃn	estimation
to evaluate	ɪˈvæljʊeɪt	évaluer
evaluation	ɪˌvæljʊˈeɪʃn	évaluation
rule	ruːl	règle
exception	ɪkˈsepʃn	exception
formula plur. *-lae*	ˈfɔːmjʊlə -lɪ	formule
to generalize	ˈdʒenrəlaɪz	généraliser
generalization	ˌdʒenrəlaɪˈzeɪʃn	généralisation
interpolation	ɪnˌtɜːpəʊˈleɪʃn	interpolation
investigation	ɪnˌvestɪˈgeɪʃn	recherche
to satisfy	ˈsætɪsfaɪ	vérifier †
to fulfil	fʊlˈfɪl	vérifier †

To satisfy : to fulfil the conditions of a given theorem, assumption, equation, etc. For example, 3 and -3 satisfy the equation $x^2 = 9$.

Subject : the term in a formula that is explicitly found by substituting values for the other variables. For example S is the subject of the formula $S = \dfrac{4\pi r^3}{3}$.
Conjecture : a suggestion based on investigation of some rule or pattern in a problem.

Geometry		
geometry	dʒɪˈɒmətrɪ	géométrie
geometric	ˌdʒɪəʊˈmetrɪk	géométrique
construction	kənˈstrʌkʃn	construction
to construct	kənˈstrʌkt	construire
plane	pleɪn	plan
point	pɔɪnt	point
line	laɪn	droite
polygon	ˈpɒlɪgən	polygone
regular	ˈregjʊləʳ	régulier
parallel	ˈpærəlel	parallèle
perpendicular	ˌpɜːpenˈdɪkjʊləʳ	perpendiculaire
ray	reɪ	demi-droite
segment	ˈsegmənt	segment
beginning point	bɪˈgɪnɪŋ —	extrémité
endpoint	ˈendpɔɪnt	extrémité
midpoint	ˈmɪdpɔɪnt	milieu
triangle	ˈtraɪæŋgl	triangle
isosceles	aɪˈsɒsɪliːz	isocèle
equilateral	ˌiːkwɪˈlæt(ə)r(ə)l	équilatéral
oblique	əˈbliːk	quelconque
scalene	ˈskeɪliːn	quelconque
vertex plur. *-tices*	ˈvɜːteks -tɪsiːz	sommet
median	ˈmiːdɪən	médiane
altitude	ˈæltɪtjuːd	hauteur
mid-perpendicular	mɪdˌpɜːpenˈdɪkjʊləʳ	médiatrice
perpendicular bisector	– baɪˈsektəʳ	médiatrice
incircle	ˈɪnsɜːl(ə)l	cercle inscrit
incentre	ˈɪnsentəʳ	centre du . . .
circumcircle	ˈsɜːkəmˌsɜːkl	cercle circonscrit

Geometry — continued		
circumcentre	ˌsɜːkəmˈsentəʳ	centre du . . .
orthocentre	ˌɔːθəʊˈsentəʳ	orthocentre
centroid	ˈsentrɔɪd	centre de gravité
angle	æŋgl	angle
degree	dɪˈgriː	degré
acute	əˈkjuːt	aigü
right	raɪt	droit
obtuse	əbˈtjuːs	obtus
angle bisector	æŋgl baɪˈsektəʳ	bissectrice
right triangle	raɪt —	triangle rectangle
hypotenuse	haɪˈpɒtənjuːz	hypoténuse
adjacent	əˈdʒeɪsənt	adjacent
opposite	ˈɒpəzɪt	opposé
trigonometry	ˌtrɪgəˈnɒmətrɪ	trigonométrie
sine	saɪn	sinus
cosine	ˈkəʊsaɪn	cosinus
tangent	ˈtændʒənt	tangente
quadrilateral	ˌkwɒdrɪˈlæt(ə)r(ə)l	quadrilatère
trapezoid	ˈtræpɪzɔɪd	quadrilatère
trapezium	trəˈpiːzɪəm	trapèze
parallelogram	ˌpæræˈleləgræm	parallèlogramme
rhombus plur. *-bi*	ˈrɒmbəs -baɪ	losange
rectangle	ˈrekˌtæŋgl	rectangle (*nom*)
square	skweəʳ	carré
diagonal	daɪˈægənl	diagonale
area	ˈeərɪə	aire
perimeter	peˈrɪmɪtəʳ	périmètre
circumference	səˈkʌmf(ə)r(ə)ns	circonférence
circle	ˈsɜːkl	cercle
semicircle	ˈsemɪˌsɜːkl	demicercle
centre	ˈsentəʳ	centre
radius plur. *-dii*	ˈreɪdɪəs -dɪˌaɪ	rayon (d'un cercle)
diameter	daɪˈæmɪtəʳ	diamètre
chord	kɔːd	corde
arc	ɑːk	arc (de cercle)

Geometry — continued		
semicircle	ˈsemɪˌsɜːkl	demicercle
annulus plur. *-li*	ˈænjʊləs -laɪ	couronne (circulaire)
ellipse	ɪˈlɪps	ellipse
oval	ˈəʊvəl	ovale
to bisect	baɪˈsekt	couper en deux moitiés
analytic geometry	ˌænəˈlɪtɪk —	géométrie analytique
coordinate	kəʊˈɔːdnət	coordonnée
coordinate geometry		géométrie analytique
abscissa plur. *-sae*	æˈbsɪsə -siː	abscisse
ordinate	ˈɔːdnət	ordonnée
quadrant	ˈkwɒdrənt	quadrant
coordinate system		repère
basis plur. *-ses*	ˈbeɪsɪs -siːz	base
orthogonal	ɔːˈθɒg(ə)n(ə)l	orthogonal
orthonormal	ɔːθəˈnɔːm(ə)l	orthonormal
normalized	ˈnɔːməlaɪzd	normé *ou* normal
axis plur. *axes*	ˈæksɪs ˈæksiːz	axe
gradient	ˈgreɪdɪənt	pente *ou* coefficient directeur
slope	sləʊp	pente *ou* coefficient directeur
transformation	ˌtrænsfəˈmeɪʃn	transformation
translation	trænsˈleɪʃn	translation
vector	ˈvektə^{r}	vecteur
sense	sens	sens
direction	dɪˈrekʃn	direction
modulus (of a vector)	ˈmɒdjʊləs	norme
magnitude (of a vector)	ˈmægnɪtjuːd	norme
length (of a vector)		norme
reflection	rɪˈflekʃn	symétrie
enlargement	ɪnˈlɑːdʒmənt	agrandissement
locus plur. *-ci*	ˈləʊkəs -saɪ	lieu

Solid geometry		
solid geometry	ˈsɒlɪd —	géométrie dans l'espace
stereometry	ˌsterɪˈɒmɪtrɪ	géométrie dans l'espace
cone	kəʊn	cône
conical	ˈkɒnɪkl	conique (*adj*)
conic section	ˈkɒnɪk ˈsekʃn	(une) conique
cube	kjuːb	cube
cuboid	ˈkjuːbɔɪd	pavé droit
cylinder	ˈsɪlɪndə^{r}	cylindre
cylindrical	sɪˈlɪndrɪkl	cylindrique
circular cylinder	ˈsɜːkjʊlə^{r} —	cylindre de révolution
edge	edʒ	arête
face	feɪs	face
frustum plur. *-ta*	ˈfrʌstəm -tə	tronc
conical frustum		tronc de cône
horizon	həˈraɪzn	horizon
horizontal	ˌhɒrɪˈzɒntl	horizontal
vertical	ˈvɜːtɪkl	vertical
polyhedron plur. *-dra*	ˌpɒlɪˈhiːdrən -drə	polyèdre[2]
tetrahedron	ˌtetrəˈhiːdrən	tétraèdre
octahedron	ˌɒktəˈhiːdrən	octaèdre
dodecahedron	ˌdəʊdekəˈhiːdrən	dodecaèdre
icosahedron	ˌaɪkəsəˈhiːdrən	icosaèdre
pictorial drawing	pɪkˈtɔːrɪəl ˈdrɔːɪŋ	dessin en perspective
perspective drawing	pəˈspektɪv —	dessin en perspective
cabinet drawing	ˈkæbɪnɪt —	perspective cavalière
plan	plæn	vue de dessus
pyramid	ˈpɪrəmɪd	pyramide
skew lines	skjuː —	droites non coplanaires
sphere	sfɪə^{r}	sphère
spheric	ˈsfɪərɪk	sphérique
spherical	ˈsferɪkl	sphérique
great circle	greɪt —	grand cercle
volume	ˈvɒljuːm	volume

[2]the following words finishing with hedron have the same irregular plural in hedra

Arithmetic		
arithmetic	əˈrɪθmətɪk	arithmétique (*nom*)
arithmetic	ˌærɪθˈmetɪk	arithmétique (*adj.*)
arithmetical	ˌærɪθˈmetɪkl	arithmétique (*adj.*)
number	ˈnʌmbə^{r}	nombre
figure	ˈfɪgə^{r}	chiffre
digit	ˈdɪdʒɪt	chiffre
Roman numeral	ˈrəʊmən ˈnjuːm(ə)r(ə)l	chiffre romain
whole number	həʊl	entier
integer	ˈɪntɪdʒə^{r}	entier relatif
directed number	dɪˈrektɪd	entier relatif
rational	ˈræʃənl	rationnel
ratio	ˈreɪʃɪəʊ	rapport
irrational	ɪˈræʃənl	irrationnel
positive	ˈpɒzətɪv	positif
negative	ˈnegətɪv	négatif
null	nʌl	nul
addition	əˈdɪʃn	addition
plus	plʌs	plus
sum	sʌm	somme
summand	ˈsʌˌmænd	terme
addend	əˈdend	terme
carry	ˈkærɪ	retenue
term	tɜːm	terme
subtraction	səbˈtrækʃn	soustraction
minus	ˈmaɪnəs	moins
difference	ˈdɪfrəns	différence
multiplication	ˌmʌltɪplɪˈkeɪʃn	multiplication
times	taɪmz	fois
multiplied by	ˈmʌltɪplaɪd baɪ	multiplié par
product	ˈprɒdʌkt	produit
factor	ˈfæktə^{r}	facteur
factorize	ˈfæktəˌraɪz	factoriser
factorization	ˌfæktəraɪˈzeɪʃn	factorisation
division	dɪˈvɪʒn	division
divisor	dɪˈvaɪzə^{r}	diviseur

Arithmetic — continued		
dividend	ˈdɪvɪˌdend	dividende
remainder	rɪˈmeɪndə^{r}	reste
unity	ˈjuːnətɪ	unité
divided by	dɪˈvaɪdɪd	divisé par
over	ˈəʊvə^{r}	sur
quotient	ˈkwəʊʃnt	quotient
multiple	ˈmʌltɪpl	multiple
divisible	dɪˈvɪzəbl	divisible
prime	praɪm	premier
H.C.F.		PGCD
highest common factor		plus grand commun diviseur
L.C.M.		PPCM
lowest common multiple		plus petit commun multiple
even	ˈiːvn	pair
odd	ɒd	impair
fraction	ˈfrækʃn	fraction
numerator	ˈnjuːməreɪtə^{r}	numérateur
denominator	dɪˈnɒmɪneɪtə^{r}	dénominateur
inverted	ɪnˈvɜːtɪd	inversée
to invert	ɪnˈvɜːt	inverser
to cancel	ˈkænsl	simplifier †
to simplify	ˈsɪmplɪfaɪ	simplifier
equals	ˈiːkwəlz	égale
is equal to	ˈiːkwəl	est égal à
inverse	ˌɪnˈvɜːs	
additive inverse	ˈædɪtɪv	opposé
multiplicative inverse	ˌmʌltɪˈplɪkətɪv	inverse
reciprocal	rɪˈsɪprəkl	inverse
squared	skweəd	carré
cubed	kjuːbd	cube
a to the power n	paʊə^{r}	a (à la) puissance n
exponent	ɪkˈspəʊnənt	exposant
index plur. *-dices*	ˈɪndeks –dɪsiːz	exposant

Arithmetic — continued		
square root of	skweə ruːt əv	racine carrée de
radical sign	ˈrædɪkl saɪn	radical
to square	skweəʳ	mettre au carré
inequality	ˌɪnɪˈkwɒlətɪ	inégalité
less than	les	$<$
less than or equal to		$\leq$
not greater than		$\leq$
greater than	ˈgreɪtəʳ	$>$
greater than or equal to		$\geq$
not less than		$\geq$
to arrange	əˈreɪndʒ	ranger
to order	ˈɔːdəʳ	ordonner
to rank	ræŋk	ordonner
bracket	ˈbrækɪt	crochet, parenthèse, accolade
brace	breɪs	accolade
(square) bracket		crochet
parenthesis plur. *-ses*	pəˈrenθɪsɪs -siːz	parenthèse
approximately equal to	əˈprɒksɪmətlɪ	$\approx$
percentage	pəˈsentɪdʒ	pourcentage
per cent	pəˈsent	pourcent %
per mil	pəˈmɪl	pour mille ‰
per mille	pə ˈmɪlɪ	pour mille
quantity	ˈkwɒntətɪ	quantité
absolute value	ˈæbsəluːt ˈvæljuː	valeur absolue
modulus -li	ˈmɒdjʊləs -ˌlaɪ	valeur absolue
pair	peəʳ	paire
ordered pair	ˈɔːdəd —	couple
standard form	ˈstændəd fɔːm	notation scientifique
surd	sɜːd	sourd [3]

To cancel : to simplify a fraction by dividing both numerator and denominator by the same number of variable which must be a common factor of both of them. For example, the algebraic expression $5x/25$ cancels to $x/5$ when divided top and bottom by 5.

[3] In French this word is obsolete in the present meaning.

Surd : an expression containing the root of a number which is not a perfect square. For example, $\sqrt{3}$.

Calculation		
rounding	ˈraʊndɪŋ	arrondi
truncating	trʌŋˈkeɪtɪŋ	troncation
approximate	əˈprɒksɪmət	approché
approximately	əˈprɒksɪmətlɪ	approximativement
to approximate	əˈprɒksɪmeɪt	approcher
scale	skeɪl	échelle
significant figure	sɪˈgnɪfɪkənt ˈfɪgə^{r}	chiffre significatif
decimal place	ˈdesɪml —	décimale
accurate	ˈækjʊreɪt	précis
accuracy	ˈækjʊrəsɪ	précision
to tabulate	ˈtæbjʊleɪt	tabuler
table	ˈteɪbl	tableau

Algebra		
algebra	ˈældʒɪbrə	algèbre
algebraic	ˌældʒɪˈbreɪɪk	algébrique
to substitute a for b	ˈsʌbstɪtjuːt	remplacer b par a
symbol	ˈsɪmbl	symbole
value	ˈvæljuː	valeur
numerical	njuːˈmerɪkl	numérique
a variable	ˈveərɪəbl	une variable
an unknown	ˌʌnˈnəʊn	une inconnue
equation	ɪˈkweɪʒn	équation
to equate a to b	ɪˈkweɪt	mettre a et b en équation
linear equation	ˈlɪnɪə^{r} —	équation linéaire *ou* du premier degré
equality	ɪˈkwɒlətɪ	égalité
inequality	ˌɪnɪˈkwɒlətɪ	inégalité ; inéquation
inequation	ˌɪnɪˈkweɪʒn	inéquation
simultaneous equations	ˌsɪmlˈteɪnɪəs	système d'équations

Algebra — continued		
coefficient	ˌkəʊɪˈfɪʃnt	coefficient
a constant	ˈkɒnst(ə)nt	une constante
expression	ɪksˈpreʃn	expression
in terms of		en fonction de
quadratic equation	kwɒˈdrætɪk —	équation du second degré
to solve	sɒlv	résoudre
solution	səˈluːʃn	solution
solution set		ensemble des solutions
to check	tʃek	vérifier †

to check : Don't forget to check your answer! Check any formulae with the formula sheet, if provided.

Analysis		
analysis	əˈnæləsɪs	analyse
function	ˈfʌŋkʃn	fonction
mapping	mæpɪŋ	fonction
to map a onto b		associer b à a
graph	grɑːf	courbe représentative
grid	grɪd	quadrillage
to plot	plɒt	placer des points†
to sketch	sketʃ	esquisser †
sketch		esquisse
increasing	ɪnˈkriːsɪŋ	(strict^{nt}) croissant
decreasing	diːˈkriːsɪŋ	(strict^{nt}) décroissant
non-increasing		décroissant (sens large)
non-decreasing		croissant (sens large)
period	ˈpɪərɪəd	période
periodic	ˌpɪərɪˈɒdɪk	périodique
affine	əˈfaɪn	affine
linear	ˈlɪnɪə^{r}	linéaire *ou* affine
parabola	pəˈræbələ	parabole
hyperbola plur. *-lae*	haɪˈpɜːbələ -lɪ	hyperbole
rectangular hyperbola	rekˈtæŋgjələ^{r}	hyperbole équilatère

Analysis — continued		
domain	dəʊˈmeɪn	ensemble de définition *ou* source
image	ˈɪmɪdʒ	image
pre-image or *preimage*	priːˈɪmɪdʒ	antécédent
counterimage	ˌkaʊntəˈrɪmɪdʒ	antécédent
argument	ˈɑːgjʊmənt	argument
turning point	ˈtɜːnɪŋ —	extrémum
maximum plur. *-ma*	ˈmæksɪməm	maximum
minimum plur. *-ma*	ˈmɪnɪməm	minimum
constant	ˈkɒnstənt	constant (adj.)
trigonometric function	ˌtrɪgənəˈmetrɪk	fonction trigonométrique
circular function	ˈsɜːkjʊləʳ	fonction circulaire
radian	ˈreɪdɪən	radian

To plot : **1** to locate or mark points (on a graph) relative to a coordinate system. **2** to draw (a curve) through these points.

Sketch : a rough drawing or graph in which the main features are marked clearly.

Statistics		
statistics	stəˈtɪstɪks	statistiques
statistician	ˌstætɪˈstɪʃn	statisticien
survey	ˈsɜːvei	enquête
to survey	sɜːˈvei	enquêter
poll	pəʊl	sondage
harmonic	hɑːˈmɒnɪk	harmonique
quadratic	kwɒˈdrætɪk	quadratique [kwa –]
mean	miːn	moyenne
root mean square	ruːt — skweəʳ	moyenne quadratique
average	ˈævərɪdʒ	moyen(ne)
bar chart	ˈbɑːˌtʃɑːt	diagramme en bâtons
bimodal	baɪˈməʊdl	bimodal(e)
class	klɑːs	classe
interval	ˈɪntəvl	intervalle
datum plur. *-ta*	ˈdeɪtəm -tə	donnée(s)

Statistics — continued		
continuous data	kənˈtɪnjʊəs —	données continues
discrete data	dɪˈskriːt —	données discrètes
frequency	ˈfriːkwənsɪ	effectif
cumulative frequency	ˈkjuːmjʊlətɪv —	effectif cumulé
dispersion	dɪˈspɜːʃn	dispersion
frequency polygon	— ˈpɒlɪgən	polygone des effectifs
histogram	ˈhɪstəgræm	histogramme
index plur. *-dices*	ˈɪndeks -dɪsiːz	index
range	reɪndʒ	étendue
quartile	ˈkwɔːtaɪl	quartile [kwa –]
line of best fit	laɪn əv best fɪt	droite d'ajustement
scatter diagram	ˈskætə^r ˈdaɪəgræm	nuage de points
mean deviation	— ˌdiːvɪˈeɪʃn	écart absolu moyen
standard deviation	ˌstændəd diːvɪˈeɪʃn	écart-type
median	ˈmiːdɪən	médiane
mode	məʊd	mode
ogive	ˈəʊdʒaɪv	polygone des effectifs cumulés (croissants)
percentile	pəˈsentaɪl	centile
pie chart	paɪ —	diagramme circulaire
population	ˌpɒpjʊˈleɪʃn	population
proportion	prəˈpɔːʃn	proportion
proportional to	prəˈpɔːʃənəl tə	proportionnel à
lower quartile	ləʊə^r —	premier quartile
upper quartile	ˈʌpə^r —	troisième quartile
sample	ˈsɑːmpl	échantillon
relative frequency	ˈrelətɪv —	fréquence
variable	ˈveərɪəbl	caractère
category	ˈkætəgərɪ	catégorie
qualitative	ˈkwɒlɪtətɪv	qualitative
quantitative	ˈkwɒntɪtətɪv	quantitative

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